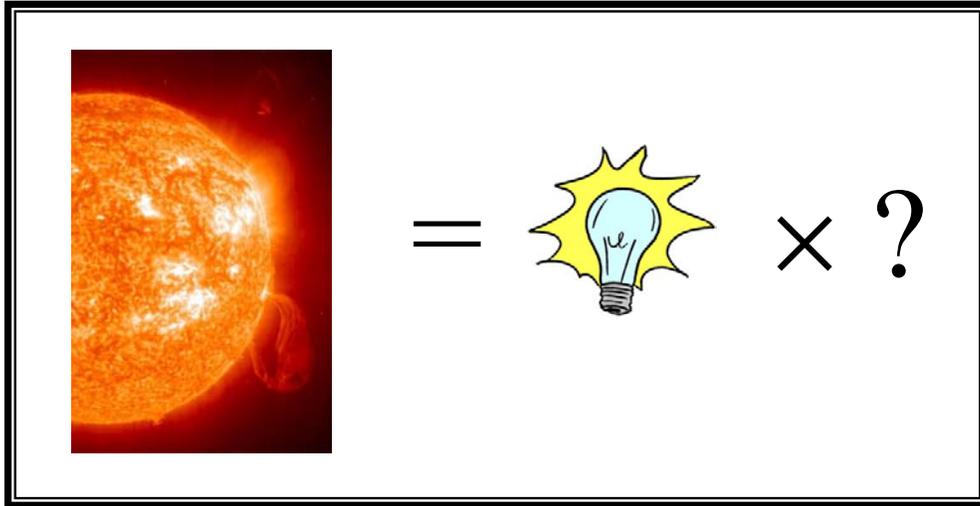


# How Many CASTLE Bulbs Would You Need To Match the Brightness of the Sun?



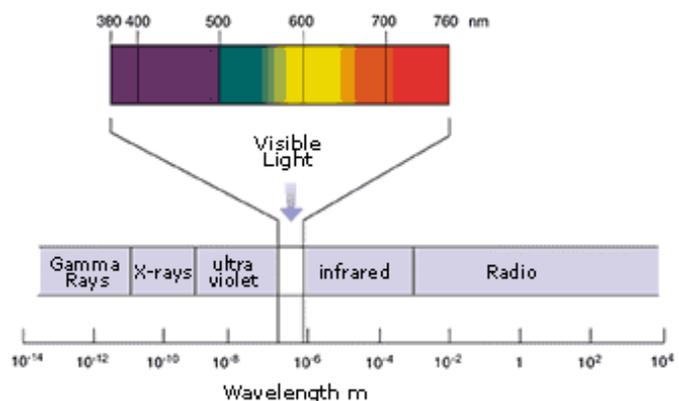
Please do all of your work **in your lab notebook**, such that it can be understood without referring back to this worksheet. That doesn't mean you should spend all of your time copying every bit of information from this worksheet! However, at every step you should explain **what** you are doing, and **why** you are doing it. Include sketches where appropriate (of your apparatus, for example).

## A. Introduction

A light looks brighter when you are close to it, and dimmer when you are further away. In this lab we will measure this effect, and use our data to determine just how bright the Sun “*really*” is. The Sun is 150 million kilometers away (93 million miles), so if we were to try to build the Sun out of light bulbs, we'd need a bunch! But, how many?

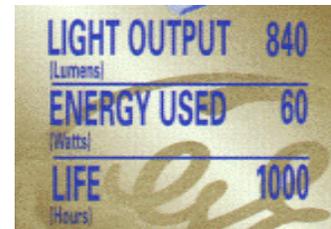
## B. Our units of measurement

In this lab, we will use light sensors to measure the brightness of two objects: the Sun, and a CASTLE light bulb. What do we mean by “brightness?” The electromagnetic (EM) spectrum is a big place, including infrared, ultraviolet, radio, and x-rays, to name a few. What we care about is how bright these sources look in **visible light**, which is to say, the part of the electromagnetic spectrum that human eyes are sensitive to.



Suppose that you have two household incandescent bulbs: one is labeled “60W,” the other “100W.” If you try them both in the same fixture, which one will make your room brighter? The number of **watts** (W) marked on a bulb gives an indication of how much energy it uses per second. Now, suppose you have a 60W incandescent bulb and a 9W fluorescent bulb. Which one makes your room brighter? It’s not what you might expect... in fact, they look about the same! The incandescent bulb emits a broad spectrum of visible and infrared radiation, while the more efficient fluorescent bulb emits only visible light. Everyone knows that an incandescent bulb gets “too hot to handle” when it has been on for a while. This is evidence that the bulb is not using all 60 watts to provide light for your room; some of the energy is being used to heat the bulb! Fluorescent bulbs tend to be much cooler to the touch.

If two light sources are the same distance away and appear equally bright to the average human eye, then we say that they have the same **luminous flux**, measured in **lumens**. The next time you replace a light bulb in your house, take a look at the packaging – it will probably tell you not only the energy usage in watts, but also the light output in lumens. The measurement of this quantity takes into account the fact that the average human eye responds differently to different colors of visible light, and is totally insensitive to the rest of the EM spectrum. Luminous flux is basically the amount of light, perceivable to the average human eye, emitted in all directions by the bulb each second.



What if the light sources are at different distances? Well, you know that a light bulb appears brighter when it is closer to you. We compare brightness at different distances by measuring a slightly different quantity: the amount of light, perceivable to the average human eye, that passes through a *unit of area* each second. This quantity is called **illuminance**, and is measured in **lux**, where 1 lux = 1 lumen/m<sup>2</sup>. The illuminance of a light source is related to the amount of light that would pass through the pupil of your eye – or through the aperture of the light sensor – every second.

Here’s the bottom line:

*The brighter the bulb looks, the higher the lux value we would measure.*

## C. “Observing” the Sun

Now we’re ready to take some data. We will point our light sensor at the Sun, and although the illuminance values won’t mean much to us yet, we’ll be able to bring them “down to Earth” at the end of class by using the experience we develop with measuring CASTLE bulbs at various distances.

You should never look directly at the Sun, as it will damage your eyes. How will we know that we are pointing the sensor correctly? The data-acquisition software *Logger Pro* will take a reading every tenth of a second, and display it on a graph of Illuminance vs Time. The Sun is the brightest thing around, so the largest value we measure will have been the one when we were pointed closest to the Sun.

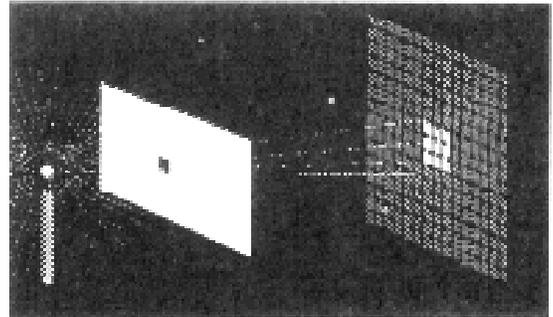
1. Estimate, from the computer plot, the true value of the solar illuminance.

*Logger Pro* can do a lot of our analysis for us. One function we will use frequently in the next few hours is the **STAT** button at the top of the screen (also available from pull-down menus as **Analyze > Statistics**). This gives us, among other things, the minimum, maximum, and average illuminance value detected while we are collecting data.

2. What is the maximum illuminance that we have detected?
3. How good was your estimate?
4. Do you think that we might get a different value if we measured at a different time of day? Why?

## D. Demonstration: Motivating the “inverse-square” law

For this exercise, we will use a Mini Mag Lite – with the reflector removed and with the bulb seen end-on – as an approximate point-source of visible light. We turn off the lights and shine the flashlight toward a 1/2” x 1/2” square hole, and allow this light to fall on a movable screen, upon which is drawn a grid of squares. (The figure at right shows the flashlight in a different orientation than we will use.)



1. Start a data table in your notebook, with the following column headings:

Number of Illuminated Squares	Distance ( $d$ ) from Source to Screen [m]	Fraction of Total Light per Square
...	...	...

2. Record the measurements. Can you see a relationship between the data in the second and third columns?

The same amount of light passes through the square hole, no matter where we are holding the screen, and so the illuminated area gets dimmer as that light gets spread out over more and more squares on the grid. If we use “1 square” as our unit for measuring areas, then our third column above gives an amount of visible light per second that hits a unit of area. The units may be different, but this was our definition of the quantity called “illuminance”!

3. Is the data in your third column consistent with illuminance being proportional to  $1/d^2$ ? This is known as the **inverse-square law** for point-sources of light.

## E. Experimenting with bulbs

Now you will take some data on your own, in groups of 2 or 3. We will have to turn off the room lights so that we are only measuring the light from the bulbs we are studying. Your group will have a flashlight, but **PLEASE**, use it only sparingly and between the light-shields provided for your group. You don't want to contaminate anyone else's measurements!

First things first: Make sure your bulb is **ON**. We want to give it time to warm up, so that its brightness will be stable once we are ready to measure it.

Each of you has experienced at least one day of the CASTLE electricity curriculum, so you already have some experience with the bulbs and battery packs that we will be using. For today's lab, the bulbs have been given just enough "juice" for them to shine with maximum light without burning out.

You will find your bulb perched atop its battery pack, which is free to slide along a meter-stick. We will be keeping the light sensor stationary, and we will be using the meter stick to measure the distance from the sensor to the bulb... so it is important that the alignment and position of the sensor be correct! It should point directly at the bulb's glowing filament, no matter where along the meter stick the bulb is. Its opening should also be directly above the "zero" end of the meter stick. Finally, the bulb's filament should be well aligned to the front edge of the battery pack, so that we can simply align the battery pack to a given position on the meter stick.

1. Were you happy with the alignments as you found them? If not, what did you change? Carefully record these changes in your lab notebook.
2. Sketch the experimental setup of your equipment in your lab notebook.

Now we begin the major task of this lab: To make illuminance measurements of our bulbs for distances from 5 – 15 cm at 2-cm intervals, and verify that these measurements are consistent with an inverse-square law.

Position the battery pack so that its leading edge is at the 5-cm line on the meter stick. Keep in mind that the distance we are attempting to measure is from the sensor to the filament (which we assume to be a point-source). All measurements have some uncertainty (or "error") associated with them. We will now consider how well we know this distance.

3. Given that you have aligned the battery pack as best you can to the 5-cm line, what do you think is the largest possible **true** distance ( $d_{max}$ ) between the sensor and the filament? The smallest possible true distance ( $d_{min}$ )?
4. What do you think might be the sources of this uncertainty?
5. Our calculations will be simpler if we measure all of our distances in *meters*. Convert  $d$ ,  $d_{max}$ , and  $d_{min}$  into meters and record your answers in your lab notebook. (100cm = 1m.)

Suppose that your largest possible distance is  $0.05\text{m} + \Delta d$ , and your smallest possible distance is  $0.05\text{m} - \Delta d$ . Then, standard notation is to write your measurement as “ $0.05\text{m} \pm \Delta d$ ”.

- Record your value of  $\Delta d$ . (This uncertainty will hold for all of your distance measurements.)  
*Don't forget the units!*

When you press the **Collect** button in the *Logger Pro* software, the program will collect 10 measurements per second for 30 seconds, and plot them on a graph, just as you saw when we were measuring the Sun. When it is done, you can press **Ctrl-U** to get a closer look at the data (you must have the graph window selected when you do this). You will notice that there is some fluctuation between the individual measurements. The measurements will pile up in distinct horizontal lines; this is because the signal from the sensor is *digitized*, and so the smallest possible difference between any two measured values is 0.2 lux.

- By eye, estimate the average illuminance (in lux) measured during this run.
- By eye, estimate the uncertainty (in lux) in this average.
- As mentioned previously, the software can do this analysis for you. Press the **STAT** button, and record the average illuminance  $I$  and its error  $\Delta I$ .
- How close were your estimates to the computer's calculations?
- In your lab notebook, start a data table with the following columns. You already have data on  $d$ ,  $I$ , &  $\Delta I$  at 0.05m, so fill in those cells now.

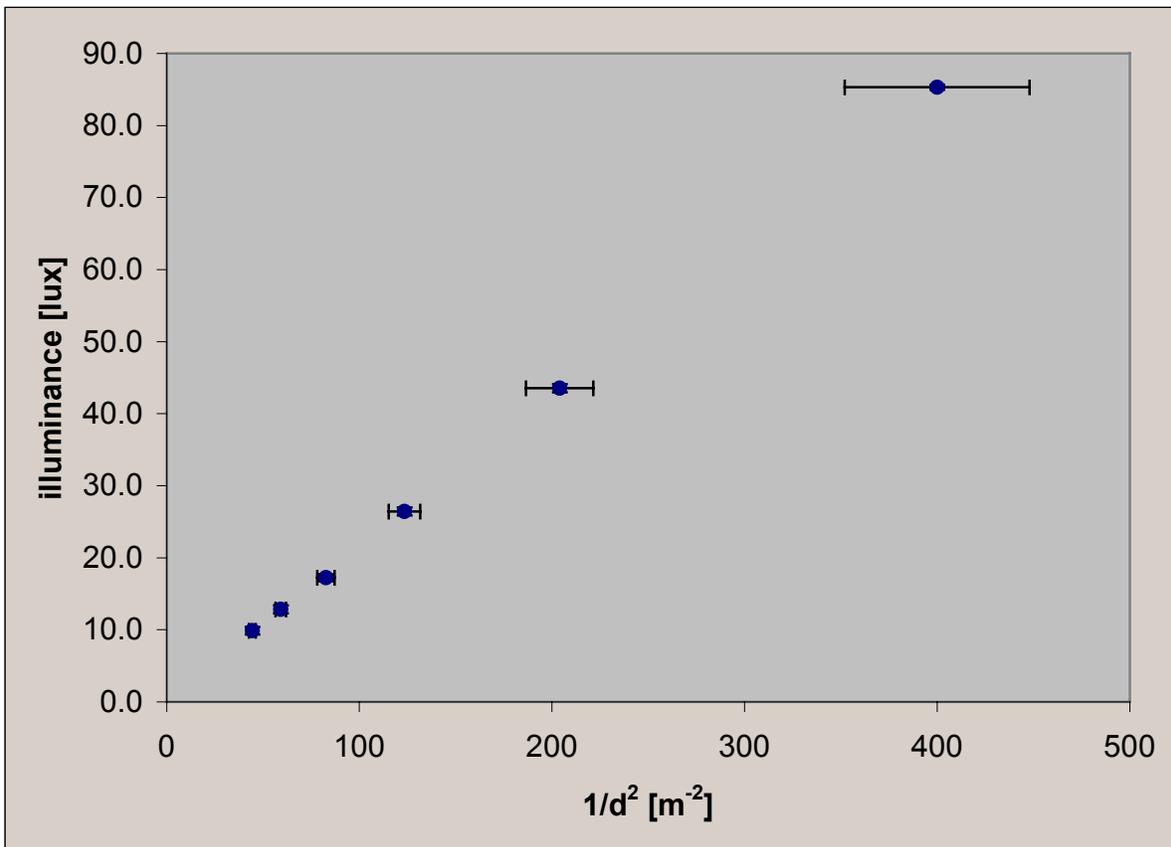
$d$ [m]	$I$ [lux]	$\Delta I$ [lux]	$1/d^2$ [m <sup>-2</sup> ]	$1/d_{max}^2$ [m <sup>-2</sup> ]	$1/d_{min}^2$ [m <sup>-2</sup> ]
...	...	...	...	...	...

You will now begin taking illuminance measurements at larger distances. Every measurement that follows will be smaller than your 5-cm measurement, and you will want to see the data as it comes in, to make sure that nothing is going wrong. (Occasionally, the sensor goes crazy and “pops up” to an unreasonably large value, or has larger-than-usual fluctuations. These errors can be fixed; see your instructor.) The easiest way to keep an eye on your data is to reload the experiment file, and avoid zooming-in during future measurements. To reload the experiment file, simply close *LoggerPro* by clicking the **X** in the upper-right of your screen, then double-click the icon named **InvSquare.MBL** on the desktop. See your instructor for assistance.

- Take similar measurements at 7, 9, 11, 13, & 15 cm. (You may take the software's word on the average and uncertainty in the illuminance; there is no need to estimate them all by-eye.) Record the results in your table.
- In your lab notebook, make a graph with  $I$  on the vertical axis and  $d$  on the horizontal. Plot your points on this graph, and connect the dots. This is what an inverse-square law looks like!

The functional form of the relationship between illuminance and distance is  $I = m / d^2$ , as per the inverse-square law. Now we seek to find a value for  $m$ . If we treat  $(1/d^2)$  as the variable along the horizontal axis, then  $m$  is just the slope of a straight line!

14. Calculate  $1/d^2$  for  $d = 0.05\text{m}$ , and record the answer in Column 4 your data table.
15. Consider how well we know the above result. We said previously that the largest and smallest possible values of the distance were  $0.05\text{m} \pm \Delta d$ , respectively. What, then, are the largest and smallest possible values of  $1/d^2$  for  $d = 0.05\text{m}$ ? Record these values in Columns 5 & 6 of your data table.
16. Complete your table by calculating  $1/d^2$ ,  $1/d_{\max}^2$ , &  $1/d_{\min}^2$  for all remaining values of  $d$ .
17. In your lab notebook, construct a plot of with  $I$  on the vertical axis and  $1/d^2$  on the horizontal axis. You'll want it to take up as much of the page as possible.
18. Plot each of the points from your data table. Also, include horizontal and vertical "error bars" for each point. That is, if the value of  $1/d^2$  is  $400 \text{ m}^{-2}$  and the range of uncertainty runs from  $356 \text{ m}^{-2}$  to  $453 \text{ m}^{-2}$ , put a dot at 400, then draw a horizontal line through the point that covers the whole range of uncertainty. Do likewise for the illuminance values. *You will be using all of the information in Columns 2-6 of your data table for each point!* A sample plot is shown below. (The vertical error bars are too small to be seen.)



19. Now, take your best guess at a “best-fit” line that goes **through the origin** and passes close to all of your points. Try to put as many of the points above your line as below it, and try to make your line pass through the error bars of at least 2/3 of all the points.
20. From the line you have drawn, determine its slope  $m$ . (Do not use the data points. The line you drew contains more information than any one point!) Don’t forget the units!
21. Take a look at the units of your slope, and recall that  $1 \text{ lux} = 1 \text{ lumen/m}^2$ . Do the units of the slope simplify? What do you think might be the physical meaning of this slope you have calculated?

Now we have a formula,  $I = m / d^2$ , that lets us predict how bright our bulb will be at any given distance! Note that, in order to use this formula, distances must be measured in *meters*.

## F. Scaling up

1. In a house with 8-foot (2.44-meter) ceilings, a light fixture is 1.5m above the tabletop. If your CASTLE bulb were somehow mounted in this fixture, what illuminance would you measure at the tabletop?
2. In scientific notation, the distance from the Sun to the Earth is  $150 \times 10^9$  (150 billion) meters. If your CASTLE bulb were that far away, what illuminance would you measure?
3. We measured the illuminance of the Sun at the beginning of class. How many CASTLE bulbs would you need to put into the ceiling fixture to obtain the same illuminance at the tabletop as the Sun would provide? How many CASTLE bulbs at the position of the Sun would you need?
4. The packaging of a 100W light bulb indicates that the bulb has a light output of 1,315 lumens. How would your answers to Question 3 change if you were to use these 100W bulbs instead of our CASTLE bulbs? (NOTE: The slope in your formula may have *units* of lumens, but in order to get a *slope* from this light output measurement, you’ll have to divide by  $4\pi$ . Thus,  $I = (1315 / 4\pi) / d^2 = 104.6 / d^2$ .)

### Image References:

- <http://antwrp.gsfc.nasa.gov/apod/ap010301.html>
- <http://gilmore-engineers.com/randd.html>
- <http://www.npl.co.uk/npl/publications/light/light4.html>
- <http://www.light-source.com/newproducts.html>
- [http://www.exploratorium.edu/snacks/inverse\\_square\\_law.html](http://www.exploratorium.edu/snacks/inverse_square_law.html)