## How Sweet It Is – How many sugar cubes do you need to fill the big dome?

What do you think? Don't worry about being wrong, just think critically and give it your best!

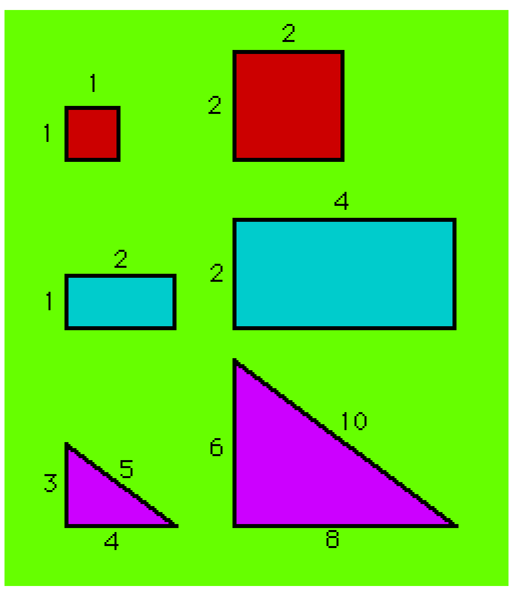
- How many sugar cubes do you think it would take to fill the large dome with sugar cubes?
- How would you know if you are right or wrong?
- How do astronomers develop models of how large or how far away something is if they can't make direct measurements?

**Procedure:**

Write the answers to question and/or problems in your laboratory notebooks

Before we being let's work on some background.

If two objects have the same shape, they are said to be geometrically similar. By definition, the ratio of any two linear dimensions of one object will be same for any geometrically similar object. This is easiest to illustrate with simple geometric shapes:




For each pair of geometrically similar shapes above, the proportions are identical. That is, the ratio of width to height of the rectangles is 2.0, and the ratio of the height to the hypotenuse of the triangles is 0.6. Notice that all of the dimensions of one can be calculated by multiplying the dimensions of the other by a constant, in these examples 2 (or 1/2). If you make a geometrically similar rectangle three times as high as the small one above, then its width will be three times as big as well, along with its diagonal measurement and perimeter, i.e. ANY linear dimension will be multiplied by the same factor.

Question

A. What are the relationships of geometrically similar shapes?

Now let's talk about some other characteristics, such as surface area and volume?

Question

B. What is the surface area of a cube that has sides that are 1-cm x 1-cm?

C. What is the volume of a cube that is 1-cm x 1-cm x 1-cm? Make a sketch of this cube and its dimensions in your notebook.

D. Now that you know the volume of a single cube let's grow that cube! Let's stack sugar cubes together to make a 2x2x2 cube. What is this the volume of this cube? Make a sketch of this cube, include its dimensions in your notebook, and label its volume.

E. Now add a third cube to the length, width and height of the last cube, so we have a 3x3x3 cube. What happens to the volume of the cube? Make a sketch of this cube, its dimensions in your notebook, and label its volume.

F. Fill in the table below with your calculations for each of the cubes:

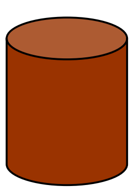
| | Small cube | Medium cube | Large cube |
|---|---|---|---|
| Length (cm) | | | |
| Surface area ($cm^2$) | | | |
| Volume ($cm^3$) | | | |
| Surface area/volume | | | |
| Surface area/length | | | |
| Volume/length | | | |

You can determine the areas and volumes intuitively by counting the exposed faces and imagining the hidden faces but how about mathematically? Devise a mathematical way to determine the m of sugar cubes you need to construct an 'n x n x n' cube with out using length (L), width (W), or Height (H) symbols in your equation. What is the formula?

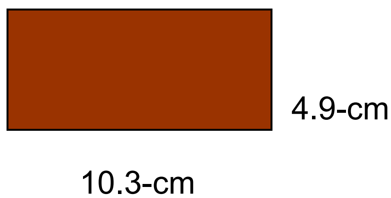
G. What happens to the surface area to volume ratio as a cube becomes bigger?

You can probably guess that mathematics can be used to find relationships between lengths, areas, and volumes of any geometrically similar objects such as cubes. However, how about other geometric shapes like cylinders and spheres?

Look at the illustration of the cylinder below.



Let's look at how a cylinder is made. It is really nothing more that a piece of paper that is rolled and taped together. If we cut out a rectangle that is 4.9-cm in height and 10.3-cm in length and roll it in the shape of the tube we can make a cylinder. These dimensions happen to be the exact size of a typical film canister.




Now let's fill the cylinder up with play sand. The sand is now occupying the volume of the cylinder, and in fact can be thought of as the volume of the cylinder. If we pour this sand into an empty film canister we will see that it fills the canister completely.

Let's talk about the dimensions of the paper that we used to construct our homemade film canister. The dimension that corresponds o 4.9cm is obviously the height of the canister. However, the dimension 10.3-cm may not be as obvious. This corresponds to the circumference of the film canister. If we wanted to find the radius of the canister we could use this information and the formula for circumference, and solve for the radius. Let's do that now

$$circumference = 2 * \pi * r$$

If we solve for the radius we have the following formula

$$radius = \frac{2 * \pi}{circumference}$$

Question

H. What is the radius of the film canister? Be sure to use the correct units!

With this information, Now make a cylinder that is double the volume of the original paper cylinder. If you have doubled the volume of the cylinder you should be able to fill your film canister up twice.

Question

H. What are the dimensions of the cylinder that has double the volume?

I. Explain how you decided on the correct dimensions.

How can we determine the volume of one of our cylinders with a mathematical formula? This is a more difficult problem --we have to know some geometry! It turns out that all geometric objects that take the shape of a circle rely on a mathematical term called pi ($\pi$). Most of you have probably heard about pi -- the number 3.14159. Pi is a very interesting number, because it has an infinite number of digits, we only rounded it off to 3.14159 but it actually goes on forever! (the closet one can approximate pi is with the fraction 355/113)To determine the volume of cylinder we need to know its height and its diameter or its radius (the radius is simply ½ the diameter). It turns out that the volume of cylinder can be found by the equation

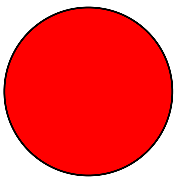
$$V = h * \pi * r^2$$

J. Find the volume of the cylinders that we have made? How many sugar cubes can you fit inside each cylinder?

If you had trouble fitting cubes of sugar into the round cylinder, you know what is meant by the old saying, “you can’t fit a square peg into a round hole.” Let’s do more investigation into how to determine the number of sugar cubes that completely fill a circular form. Take the circle that I have given to you and see how many sugar cubes will fit inside it.

You can see that this is a more difficult question, because the two objects (cubes and cylinders) are not geometrical similar.  And as such you have gaps where sugar cubes don't completely fill the circle. However, they both have volume and as such we can express the magnitude of their volume with mathematics.

Now let's look at one last geometric shape, a sphere.



A sphere is a 3-dimension object, and is commonly called a ball.

Question

K. What are the dimensions of a sphere?

This may have been a very difficult question for you, since a sphere is such a deceptively difficult object to measure and a difficult object to make. However, you should have been able to guess that it must have something to do with pi! Since we can't easily make hollow spheres and fill them with sand to determine their volume, we will simply give you the formula for determining the volume of a sphere, and remind you that any geometric object as a mathematical relationship that describes its volume and area. It turns out that the volume of a sphere is given by the equation:

$$\frac{4}{3} * \pi * r^3$$

Question

L. Now, what happens to the volume of the sphere when you double the radius? How can you explain this?

Now think of the building that houses the 40" telescope – the big dome. What is the shape of the building? Study the picture below and draw the individual geometric shapes that comprise the building that houses the large telescope.

Draw the shapes in your lab notebook

Question

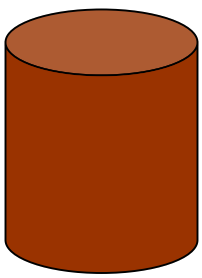
M. What do you have to know in order to determine the volume of the big dome?

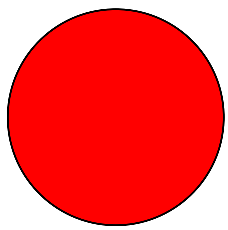
N. How many sugar cubes would it take to fill the building and the dome? Hint: think about how we determined the number of sugar cubes to completely fill our circle, and how you would fill the half sphere.

***Part II***

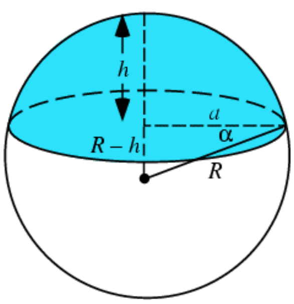
Mathematics of the Dome

You can think of the dome as consisting of two parts; a large cylinder and a hemisphere at the top. A cylinder has the shape of a large paper roll, and a hemisphere is just a ball, cut in half. To make a model of the dome, all we would have to do is find a paper roll, and cut a ball about in half and place the two together.





We know that the volume of a cylinder is given by the formula $\pi * r^2 * h$; where r is the radius of the cylinder and h is the height of the cylinder before you reach the part of the dome that looks like a hemisphere.




The volume of the hemisphere can be found by using the formula for a sphere and divided it by 2.

(http://mathworld.wolfram.com/SphericalCap.html)

To actually calculate the volume of the hemisphere gets a bit tricky unless you know calculus. The volume is really the summation of a bunch of really thin disks sitting on top of each other of decreasing size. So we will use ½ the volume of a sphere.

$$v = \frac{4/3 * \pi * r^3}{2}$$

To find the total volume of the dome all we have to do is add the volume of the cylinder and the volume of the hemisphere together.

We are in luck, because we have plenty of architectural drawings and people around who happen to know the dimensions of the dome, so we won't have to go climbing around with meter sticks!

## Materials:

A. O1-box of sugar cubes per group of students
B. Large pieces of paper to build cube structures on top of.
C. Metric rulers (30.5-cm)
D. Marking pens
E. Spherical domes (you can cut a hollow ball in half, or purchase the clear plastic domes used to measure the suns movement).
F. Calculators.

### ***Procedure:***

1. Determine the volume of one sugar cube.
2. Explain how the volume changes when you measure two cubes, three cubes, and so forth.
3. Determine the volume of the dome using the mathematics we learned today.
4. Compare the volume of the sugar cube to that of the dome and calculate the number of sugar cubes that it would take to completely fill the dome!
5. Explain how this activity has anything to do with how astronomers use mathematics, scale, modeling building, and estimate to help solve the mysteries of the universe.

### ***Some Considerations***

- When we find the dimensions of the dome we will have to be sure to convert all of them into the SI system.
- We have to measure the volume of the sugar cube in the SI system.

Metric conversions

1 inch = 2.54-cm
1 foot = 30.48-cm
100-cm = 1-meter
100-cm$^3$= 1-meter$^3$

**Teacher Notes:**

*How Sweet It Is -- How many sugar cubes will it take to fill the large dome?*

Upon completion of this activity students will be able to:

1. Explain that objects and relations in geometry correspond directly to objects and relations in algebra (e.g., volume)
2. Explain that models and scale are used in science to predict and scale up to full versions of scientific applications.
3. Explain how complex geometric objects are made of multiple simple geometric shapes.
4. Explain how to determine the volume of a single sugar cube and how to determine the large dome.
5. Estimate and explain the error associated with determining how may sugar cubes it would take to fill the dome