THE COSMOLOGICAL CONSTANT $\Lambda$

OR ENERGY DENSITY OF THE VACUUM

HAS DIMENSIONS

$\Lambda \propto \frac{\text{ENERGY}}{\text{VOLUME}}$

IN THE ABSENCE OF GRAVITY
WE DON'T CARE ABOUT THE VACUUM ENERGY, WHICH IS SIMPLY A DULL CONSTANT

IN GENERAL RELATIVITY THE VOLUME OF SPACE CAN CHANGE
AND $\Lambda$ IS MEANINGFUL
The cosmological constant or energy density of the vacuum has dimensions

\[ \Lambda \sim \frac{\text{energy}}{\text{volume}} \]

In the absence of gravity we don't care about the vacuum energy, which is simply a dull constant.

In general relativity the volume of space can change and \( \Lambda \) is meaningful.
THE COSMOLOGICAL CONSTANT $\Lambda$

OR ENERGY DENSITY OF THE VACUUM

HAS DIMENSIONS

$\Lambda \sim \frac{\text{ENERGY}}{\text{VOLUME}}$

IN THE ABSENCE OF GRAVITY
WE DON'T CARE ABOUT THE
VACUUM ENERGY, WHICH IS
SIMPLY A DULL CONSTANT

IN GENERAL RELATIVITY THE
VOLUME OF SPACE CAN CHANGE
AND $\Lambda$ IS MEANINGFUL
Just in classical general relativity, this distance scale isn't strange as there is nothing to compare it to.

But in the rest of physics there are distinguished length scales like the sizes of atoms

\[ \sim 10^{-8} \text{ cm} \]

and clearly relative to this a cosmic constant length scale

\[ 10^{28} \text{ cm} \] is anomalously big and requires some explanation.
The problem is much sharper in the presence of quantum mechanics, without which we can't sensibly discuss the atoms classically. The cosmological constant, which appears as follows in the Einstein-Hilbert action

\[ L = \frac{1}{8\pi G_N} \int d^4x \sqrt{g} \ R + \int d^4x \sqrt{g} \Lambda \]

Can just be erased if we don't like it.
That is more or less what Einstein did after it was observed that the universe was expanding.

Quantum mechanically we have to calculate the energy in the vacuum, taking account of such things as the zero point fluctuations of the electromagnetic field.
Formally, the contribution of these fluctuations to the vacuum energy is \( \frac{1}{2} \sum_{\text{modes}} \hbar \omega = \sqrt{\int \frac{d^3k}{(2\pi)^3} \frac{\hbar c}{|k|}} \) where we include all modes of wavelength \( > a \) including photons of energy up to, say, 10 GeV where electrodynamics is well tested. We get \( \Delta \lambda \sim (10 \text{ GeV})^4 \).
i.e. A CONTRIBUTION TO THE
COSMOLOGICAL CONSTANT THAT
IS TOO BIG BY A FACTOR
\[ \gg 10^{40} \]

SO SOME SORT OF VERY
PRECISE SUPPRESSION OR
CANCELLATION IS NEEDED, TO
EXPLAIN WHY THE COSMOLOGICAL
CONSTANT IS VERY SMALL IN
PARTICLE PHYSICS UNITS, OR
PERHAPS ZERO.
The question seems perplexing because of the following facts:

1. A solution seems to require a low energy mechanism to cancel, for example, the contributions of soft photons.

2. The known "effective field theory" framework for low energy physics seems not to give an answer.

3. It is very hard to change that framework in a sensible way.
THIS PUZZLE HAS BEEN WITH US FOR A LONG TIME.
"SOMETHING ELSE" SEEMS TO BE NEEDED AND ONE POSSIBILITY
ELOQUENTLY PRESENTED BY WEINBERG—
IS THAT THE SMALLNESS OF
A DOESN'T HAVE A CONVENTIONAL
SCIENTIFIC EXPLANATION BUT
MUST BE INTERPRETED
"ANTHROPICALLY"...
On the plus side, this view gave the only real theoretical prediction of $\#0$ (as opposed to seeing hints in data) before the apparent discovery;

To me this is outweighed by the minus side—such a retreat from the quest for a scientific explanation could have been offered at many junctures in the past; but in the past, science has always prevailed.
The other possibility, given the apparent inadequacy of low energy effective field theory, is that a more precise framework of physics is needed. In fact the question of why the vacuum energy is so small combines quantum mechanics and gravity; thus one may suspect that to solve the problem requires a quantum theory, for which at least for now our only candidate is string theory....
STRING THEORY, APART FROM FORCING QUANTUM GRAVITY UPON US AND UNIFYING IT WITH MATTER AND OTHER FORCES, HAS NO FUNDAMENTAL DIMENSIONLESS PARAMETER - ANALOGOUS TO

\[ \alpha = \frac{e^2}{\hbar c} \]

IN THE FUNDAMENTAL EQUATIONS

THIS IS BOTH
GOOD AND BAD

GOOD PREDICTIVE IN PRINCIPLE
BAD HARD TO COMPUTE IN PRACTICE
Thus if string theory would lead to a uniquely preferred vacuum state, then it would determine a unique value for the vacuum energy.
EVEN THE FIRST HALF OF
THIS IS WAY BEYOND WHAT WE
UNDERSTAND NOW....

TO INTERPRET THINGS IN AN
OPTIMISTIC LIGHT, I'D MAKE
A ROUGH ANALOGY WITH
THE QUANTUM MECHANICS OF
BLACK HOLES:
There are a lot of puzzles about quantum mechanics of black holes and

\[
\langle f | H | i \rangle
\]
\[
\neq 0
\]

\[
\langle i | H | f \rangle
\]

Though one might hope low energy effective field theory would suffice to answer these questions, it doesn't seem to be so
The questions again combine quantum mechanics and gravity so one might think a theory such as string theory that combines them is needed.

None of this happened for a long time, but finally in the mid-1990s – 25 years after the beginnings of string theory – string theorists did enjoy some success here.
Perhaps something like this will eventually occur for the cosmological constant.

—I hope so!

The question is actually a pressing one for string theorists since—at least in my judgment—it is the key obstacle to making the models of particle physics derived from string theory more realistic...
TO EXPLAIN BETTER WHAT I MEAN HERE, I'D LIKE TO RETALK THE SITUATION THAT EXISTED IN THE EARLY 1980'S...

FROM STRING THEORY ONE COULD DERIVE CONSISTENT THEORIES OF QUANTUM GRAVITY INTERACTING WITH MATTER, BUT THE PARTICLE PHYSICS DIDN'T COME OUT RIGHT....
One could worry about various details, but what was really wrong qualitatively was that, in the models that appeared to exist at that time, the weak interactions had to conserve parity.

It is always important to focus on the key point and at that time this was it.
Then in 1984 came the Green-Schwarz anomaly cancellation followed quickly by many other developments, notably the construction of the heterotic string.

Not only was the issue of parity violation solved; the models of particle physics became much nicer, with much more elegant models of the quark and lepton gauge couplings and family structure from a unified theory that included gravity.
In post-1984 models derived from string theory, the qualitative aspects of the standard model of particle physics can be derived neatly, but we don't have much luck with the quark and lepton masses or indeed anything that depends on supersymmetry breaking.

Supersymmetry

\[ \leftrightarrow \]

New symmetry bosons/fermions
THE KEY TO MAKING A BETTER MODEL MUST BE TO FIND A GOOD MECHANISM FOR SUPERSYMMETRY BREAKING.

WITHOUT SUPERSYMMETRY BREAKING, THERE IS NO TROUBLE TO GET A STABLE VACUUM WITH ZERO COSMOLOGICAL CONSTANT.

\[ \sum_{\text{bosons}} \frac{1}{2} m^2 \phi^2 - \sum_{\text{fermions}} \frac{1}{2} m^2 \psi^2 \]
But supersymmetry breaking can very easily lead to an unacceptably big cosmological constant... \[ > 10^{40} \]

And that is what happens with the known mechanisms of supersymmetry breaking in string theory.
SO - IN TERMS OF WHAT WE UNDERSTAND NOW - STRING THEORY FAILS BECAUSE OF THE COSMOLOGICAL CONSTANT - IT IS THE ONLY THEORY THAT SO FAILS ....

IN THAT SENSE THE COSMOLOGICAL CONSTANT PROBLEM IS PARTICULARLY SEVERE FOR STRING THEORISTS ....
WHERE PROGRESS ACTUALLY\(^2\)
HAS BEEN MADE IN THE LAST
FEW YEARS ... THOUGH NOT THE
PROBLEM ANYONE I KNOW WOULD'VE
PUT AT THE TOP OF THE LIST
... IS IN UNDERSTANDING THE
MATHEMATICAL PHYSICS OF A
WORLD WITH NEGATIVE COSMOLOGICAL
CONSTANT (MALDAEVA,...)
This is a strange world with a boundary at spatial infinity.

\[ t = 0 \]

\[ \text{Spatial Infinity} \]

\[ \text{Minkowski} \]
\[ \Lambda < 0 \]

Anti de Sitter \( \text{Ads} \)

A boundary condition at spatial infinity is needed... signals can be exchanged to and from the boundary....
TWO EQUIVALENT THEORIES

GAUGE THEORY

GRAVITY
AdS SPACE

GAUGE THEORY IN d DIMENSIONS
AND GRAVITY IN d+1 DIMENSIONS

= MANY NEW INSIGHTS ON
QUESTIONS RANGING FROM
QUARK CONFINEMENT TO
BLACK HOLES
THE MATHEMATICAL PHYSICS OF A WORLD WITH POSITIVE COSMOLOGICAL CONSTANT IS ALSO STRANGE BUT SO FAR MUCH LESS UNDERSTOOD.

THE ANALOG OF MINKOWSKI SPACE IS DE SITTER SPACE

\[ ds^2 = R^2 (d\tau^2 - \sum \cosh^2 t \, d\Omega^2) \]

\[ R = 10^{28} \text{ cm} \, ? \]

EXPONENTIAL EXPANSION - (INFLATION) - FOR \( t \gg 0 \)
Because of inflation, every observer experiences a cosmic "horizon"

= \textit{Bekenstein-Hawking-Gibbons entropy}

\[ S \sim \frac{R^2}{G_N} \quad (\sim 10^{120} \text{ in } \text{"real world"}) \]

Conjecturally - finite dimensional \textit{quantum Hilbert space with} \textit{E}^s \text{ states}
THIS IS A SURPRISING CLAIM, SINCE IN PERTURBATION THEORY IT SEEMS CLEAR THAT THE HILBERT SPACE IS INFINITE-DIMENSIONAL

INFINITELY MANY MODES OF SHORT WAVELENGTH!
There has been much less progress with this than with black holes or the case \( \Lambda < 0 \) ....

Partly because there isn't any simple stringy model of de Sitter space.... (while there are such models for black holes and \( \Lambda < 0 \)) ....

Note that a simple stringy model would have \( \Lambda \) much too big....
While awaiting a really good idea about why $\Lambda$
is so small or zero, we'd really like to know what the observed phenomenon represents.

A cosmological constant is the most straightforward explanation for acceleration of the Hubble expansion.
A LARGER CLASS OF PHYSICAL MODELS THAT GIVE NEGATIVE PRESSURE AND COSMIC ACCELERATION HAVE BEEN DUBBED "QUINTESSENCE"... STONHAUER ET AL.

1. ROLLING SCALAR FIELD

\[
V(\phi) \quad \text{ROLLING}\]

SCALAR FIELD \( \phi \)

\[
V(\phi) \quad \text{NOW HERE}
\]
Such an instability is similar to what we actually get in many string models, but milder.

\( \Phi \) \& a fifth force?

Deviation from general relativity?

Test equivalence principle?

(Carroll et al.)

(in a recent version - Steinhardt & Turok - the true \( \Lambda \) is negative)
\[ \hat{\text{true}} \sim -(10^{10} \text{ GeV})^4 \]

which is small enough to be plausible in some known supersymmetry-breaking models. But the near-flatness of the potential at the values of $\phi$ in today's world looks hard to explain.
A more conservative version uses a pseudoscalar axion.

\[ V(\phi) = \lambda (\cosh(\phi/F) + 1) \]

If \( F \) is a very big mass.

Today's vacuum energy will ultimately decay to axions.

No trouble with equivalence principle.
To me this is the most conservative quintessence model that works without obvious particle physics trouble except the lack of understanding of why the true cosmological constant is so small or zero.
I conclude with some more exotic models

Spergel et al.

3. A network of cosmic strings or domain walls

\[ P = -\text{density} \]

\[ \text{Pressure} = -\frac{1}{3} \cdot \text{density (strings)} - \frac{2}{3} \cdot \text{density (domain walls)} \]

Frustrated strings
AND SOMETHING COMPLETELY DIFFERENT: CSAKI, KALOPER, TERNING INTERPRETED DIMMING OF DISTANT supernovas via photon-axion mixing. 1) circular 0 axions

ELEGANT EXPLANATION OF MAGNITUDE OF supernova DIMMING - AT A PARTIQUE PHYSICS PRICE

BUT DEFFAYET et al. suggested THE MIXING IS SPOILED BY PLASMA EFFECT OF INTERGALACTIC MEDIUM
Anyway, regardless of how the present observations are ultimately interpreted, the vanishing or extreme smallness of the cosmological constant is one of the strangest unexplained observations and probably one of our best clues...