Multiplexed Readout of Superconducting Bolometers for Cosmological Observations

by

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Abstract

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We describe the design, development, and demonstration of frequency-domain multiplexed readout technology for superconducting transition-edge sensors (TESs). This technology enables us to deploy large-scale bolometer arrays for ultra-sensitive observations of the cosmic microwave background (CMB).

Each sensor is biased with a sinusoidal voltage at a unique frequency. As the sensor impedance changes due to absorbed radiation, it amplitude-modulates its bias. The modulated sensor currents are summed and measured with a single superconducting quantum interference device (SQUID) array. The 100-element SQUID array is operated with custom shunt feedback electronics that have a bandwidth of 1MHz and a slew rate of $1.2 \times 10^7 \Phi_0/s$. Separate warm demodulation electronics recover the sky signal from each sensor.

Our eight-channel demonstration of readout multiplexing showed a multiplexer readout noise of $< 7 \text{ pA}\sqrt{\text{Hz}}$, well below the sensor noise of $15 \text{ pA}/\sqrt{\text{Hz}}$. Crosstalk between multiplexed channels is well below our design limit of 1% and demodulated noise spectra from multiplexed sensors show the expected sensor noise levels and are white at frequencies down to 200 mHz.
The current frequency-domain multiplexing technology will be used on several experiments designed to observe the CMB: The South Pole Telescope, PolarBear, and EBEX. Extensions to the technology are described that will extend the performance of the system and allow multiplexing of one hundred detectors.

Professor Adrian Lee
Dissertation Committee Chair
to my parents...
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Chapter 1

Introduction

1.1 Overview

Over the last few decades, cosmology has become a precise science that can be rigorously tested. This revolution is occurring in large part because dramatic advances in technology are making possible very sensitive observations. These observations allow us to directly test models of the expansion history of the universe, test models of the formation of the structure that we currently observe and measure the fractional amounts of baryonic matter, dark matter, and dark energy in the universe.

Measurements from a wide variety of experiments, including large-scale galaxy surveys [8], supernovae type 1a light curves, primordial nucleosynthesis abundances, and the anisotropy of the cosmic microwave background (CMB) radiation all point towards a remarkably consistent model: we live in an universe that began in a state of extremely high temperature and density. It subsequently underwent a period of rapid expansion (inflationary period) during which we hypothesize that density fluctuations were generated. Following the inflationary period was a period of expansion in which the baryonic matter of the universe was a plasma (ionization period) in thermal equilibrium with radiation and undergoing acoustic oscillations. After the ionization period, a period of expansion began during which the
baryonic matter became neutral and began collapsing under gravity to form the structure in the universe we observe today.

Measurements of the CMB in particular have yielded enormous amounts of information about our universe and provided one of the fundamental pieces of evidence for the Big Bang model described in the previous paragraph. The CMB is an electromagnetic radiation field present everywhere in the universe and first detected in 1965 [9]. It is hypothesized to be primordial radiation produced by the Big Bang: it has been massively red-shifted by the expansion of the universe and is currently at a temperature of 2.73K.

The value of measurements of the CMB radiation rests in the fact that this radiation essentially ceased interacting with baryonic matter at the very early time of only 400,000 years after the Big Bang. Except for a small fraction that have undergone secondary scattering, most CMB photons have been free-streaming from this time of “last-scattering” and thus the CMB is a gold mine of information about the state of the universe at this time. By measuring the temperature, anisotropy, and polarization characteristics of the CMB, we can characterize the composition and dynamics of the universe at this very early time.

1.2 Measurement History

1.2.1 First Detection

CMB measurements have a rich history with experimental advances are intricately entwined with technological advances. The CMB was first detected in 1964 by Penzias and Wilson as an isotropic "excess noise" at an effective temperature of $3.5 \pm 1.0 \, K$ when commissioning a new radio telescope operating at $\lambda = 7.35\,\text{cm}$. It was almost immediately identified as relic radiation from the Big Bang [10].
1.2.2 CMB Spectrum

Initially, the CMB measurements that followed its detection used radio telescopes and focused on the long-wavelength or Rayleigh-Jeans part of the CMB spectrum. Specifically, these experiments used heterodyne receivers that chopped between a cold load and the sky and measured the CMB at wavelengths < 3 mm. The results suggested that the CMB spectrum was in close agreement with that of a blackbody [11]. However, radiometer measurements at shorter wavelengths, to confirm the Wien tail of the CMB spectrum, were much more challenging. Radiometers begin to approach quantum noise limits at these shorter wavelengths and a new strategy was needed for this measurement.

Woody and Richards ([12]) were the first to successfully address this challenge. They constructed a pioneering experiment consisting of a balloon-borne bolometer-based telescope and a Fourier transform spectrometer to provide spectral discrimination for the broad-band bolometer. Their experiment measured the spectrum of the CMB across a decade of wavelengths that spanned the Rayleigh-Jeans and Wien portion of the CMB spectrum: 4 mm to 0.5 mm. They were the first to confirm the expected reduction in the spectral intensity at these shorter wavelengths and show that the CMB spectrum is in good agreement with that of a black body.

The FIRAS (Far Infrared Absolute Spectrophotometer) instrument on the COBE (Cosmic Background Explorer) satellite has provided the best measurement of the CMB spectral intensity between the wavelengths 5 mm and 0.5 mm ([13]). The spectrum agrees with a black-body of temperature $2.726 \pm 0.01$ K to a high precision (the Compton $y$ parameter, a measure of the deviation from a black-body spectrum, is only several parts in $10^5$).

1.2.3 CMB Temperature Anisotropy

The early measurements of the CMB showed that it was remarkably isotropic. The first deviation from isotropy discovered was the CMB dipole at a level of 3.3 mK. [14] measured
the dipole at a wavelength of 9.1 mm with an aircraft-borne radiometer. Subsequent measurements produced evidence for anisotropy at smaller levels and smaller angular scales. The DMR (differential microwave radiometer) instrument, also flown on the COBE satellite, unambiguously showed a temperature anisotropy in the CMB. DMR measured this anisotropy at about $10^{-5}$ and mapped it across the entire sky down to angular scales of $7^\circ$ ([15]).

In addition to the COBE satellite, the early 1990s saw a dramatic improvement in both bolometer sensitivity and HEMT (high electron mobility transistor, used as amplifiers in coherent receivers) performance. Many experiments exploited these developments to characterize the CMB anisotropy with higher sensitivity and at smaller angular scales. Two balloon-based bolometer experiments, MAXIMA ([16], [17]) and BOOMERANG ([18]), and a ground-based microwave interferometer, DASI ([19]), made partial sky maps of the CMB anisotropy at angular scales down to about $1^\circ$. A ground-based bolometer experiment, ACBAR ([20]), and a ground-based interferometer, CBI ([21]) made maps to smaller angular scales. Again, a satellite mission, WMAP (Wilkinson Microwave Anisotropy Probe, [22]), has provided the most sensitive measurement of the anisotropy of the CMB across the entire sky at angular scales down to $10^\prime$.

### 1.2.4 CMB Polarization Anisotropy

One of the most active current areas in CMB observation is the characterization of its polarization anisotropy. The scientific motivation behind these observations is summarized in chapter 2. Two experiments, DASI ([23]) and WMAP ([24]), have made low signal-to-noise detections of polarization anisotropy on $1^\circ$ scales. The largest polarization anisotropy signals are over an order of magnitude smaller than the temperature anisotropy ($1.5 \, \mu K$). Since these polarization signals are so small, to precisely characterize the CMB polarization anisotropy the next generation of experiments will need a large increase in sensitivity.
1.3 Motivation for Multiplexing

Single detectors (both bolometers and HEMTs) are starting to achieve noise equivalent temperatures (NETs) that are comparable or less than the expected photon noise limit. Thus, the sensitivity of these detectors for CMB observations is approaching a fundamental limit. The strategy for the next generation of experiments to achieve a large increase in sensitivity is deploying arrays of hundreds to thousands of detectors.

Although there are trade-offs between different detector technologies, which will be discussed in more detail in chapter 3, large arrays of bolometers are emerging as an attractive approach. In particular, the voltage-biased, transition-edge sensor (TES) bolometer, with strong electro-thermal feedback, is a promising and maturing detector technology that provides uniform parameters and controlled operating points across large arrays [25]. Furthermore, arrays of TESs can be produced monolithically with photolithographic fabrication techniques [26]. A key remaining challenge in the development and deployment of large arrays is their readout.

To achieve the required sensitivity, bolometric detectors are typically operated at a temperature below 1K (MAXIMA, for example, operated at 100 mK). Readout of large arrays of cryogenic detectors is a major instrumental challenge. In order to reduce the sheer wiring complexity of an experiment, the cost of per-channel readout components, and the heat load delivered to the coldest stages by readout wiring, the readout of these arrays will be multiplexed.

There are two complementary techniques for the multiplexed readout of TES bolometers: time-domain and frequency domain multiplexing ([6]). In time-domain multiplexing (TDM), a single readout channel is sequentially switched between each member of a set of $n$ detectors. TDM has been developed at NIST [27] and will be used for the readout of several instruments, including SCUBA-II.
In frequency-domain multiplexing (FDM), detectors are sinusoidally biased and the sensors are thus well-separated in frequency space. A single readout channel simultaneously measures the signals from a set of \( n \) detectors. The development of FDM has been done at Berkeley ([7]) and is the subject of this thesis. Two other groups are independently developing FDM systems: a group at ISAS in Japan [28] and a collaboration between SRON and VTT (Finland, The Netherlands) [29]. The Berkeley collaboration was the first to publish results from a working demonstration of an FDM system and is currently leading the other groups in integrating the system into several instruments.

We will describe the design, development, and demonstration of FDM in detail. Section 1.4 describes the overall structure of this thesis.

### 1.4 Outline

Chapter 2 will explain in more detail the scientific motivation for studying the CMB. In chapter 3, we will discuss the trade-offs between detector technologies and explain the operation of transition-edge sensor bolometers. Chapter 4 will discuss TDM and FDM, discuss the reasons for the independent development of FDM, and describe the design of the FDM readout. The superconducting quantum interference device (SQUID), the cryogenic amplifier at the core of the FDM readout, and the custom electronics developed to read out the SQUID are presented in chapter 5. Chapter 6 describes the development of high-Q cold filtering and sinusoidal biasing, two of the key components of FDM readout. Chapter 7 describes the results of the eight-channel test of FDM performed at Berkeley. Finally, in chapter 8, we discuss several extensions to the FDM technology currently under development.
Chapter 2

Science

In this chapter, we discuss the scientific motivation for observing the CMB. The standard model of the expansion of the universe is presented. The origin of the CMB and its temperature and polarization anisotropies are discussed and we explain the cosmological parameters that can be constrained by the characterization of these anisotropies.

2.1 The Isotropic and Homogeneous Universe

Observational evidence in cosmology over the last century has confirmed several remarkable properties of the observable universe: on large enough scales the universe is both isotropic and homogeneous. In addition, the universe is expanding. The isotropy and homogeneity of the universe then demand that the universe is undergoing a uniform expansion. That is, every observer, at a given time, measures the same rate of expansion. Robertson and Walker showed that with only the assumption of isotropy and homogeneity, the metric of an expanding universe with arbitrary geometry is [30]:

\[ ds^2 = c^2 dt^2 - R^2(t) \left( \frac{dr^2}{1-kr^2} + r^2( d\theta^2 + \sin^2 \theta d\phi^2 ) \right) \]  

(2.1)

where \( r \) measures the comoving distance between points, \( t \) is the proper time measured by an observer expanding with the universe, \( k \) is a measure of the geometry of the universe
(k = 0 for a flat universe, k = +1 for a spherical universe, and k = -1 for a hyperbolic universe), and \(R(t)\) is a dimensionless time-dependent scale factor that encodes the history and dynamics of the expansion of the universe. The scale factor and comoving distance are defined so that the scale factor is unity at the present time.

The power of the Robertson-Walker metric shown in equation 2.1 is its generality: the physics governing the dynamics of the universe is entirely contained in \(R(t)\). In particular, whether we use the Newtonian theory of gravity or general relativity, equation 2.1 tells us how to measure distance and time in an expanding, isotropic, and homogeneous universe.

Einstein’s field equations for general relativity allow us to describe \(R(t)\). Friedmann provided the first explicit solution of these equations:

\[
\dot{R} = \frac{8\pi G \rho}{3} R^2 - \frac{k c^2}{R_c^2} + \frac{1}{3} \Lambda R^2
\]

(2.2)

where \(R\) is the scale factor, \(\rho\) is the density of the universe, \(R_c\) is the radius of curvature of the universe, and \(\Lambda\) is Einstein’s cosmological constant. The Friedman Equation can be rewritten in a simpler form:

\[
\left(\frac{\dot{R}}{R}\right)^2 = H_0^2 \left(\Omega_m R^{-3} + \Omega_r R^{-4} + \Omega_\Lambda + \Omega_k R^{-2}\right)
\]

(2.3)

where \(H_0\) is the Hubble constant (the present rate of expansion), \(\Omega_m\) is the density of matter, \(\Omega_r\) is the density of radiation (including all relativistic matter species), \(\Omega_\Lambda\) is the energy density of the cosmological constant, and \(\Omega_k\) is curvature of the universe (expressed as an energy density). By construction, the sum of the densities is 1:

\[
\Omega_m + \Omega_k + \Omega_\Lambda = 1
\]

(2.4)

The remarkably simple Friedman equation (equation 2.3) allows us to use the matter and energy content of the universe to calculate the dynamics of the expanding universe. From this equation, it is straightforward to see that the qualitative behaviour of the universe changes during different epochs: at early times, when the scale factor was much smaller, the expansion of the universe was dominated by radiation:
\[ R(t) = (2H_0t)^{1/2} \]  

(2.5)

There was a short period of time when matter and radiation contributed comparably to the expansion of the universe, after which the universe became matter dominated:

\[ R(t) = \left( \frac{3}{2}H_0t \right)^{2/3} \]  

(2.6)

In the presence of a non-zero cosmological constant \( \Lambda \), \( \Omega_\Lambda \) dominates the dynamics of the universe at late times (i.e. large scale factor) and the expansion is exponential:

\[ R(t) = R_0 e^{H_0t} \]  

(2.7)

### 2.2 Cosmic Microwave Background

One of the most convincing pieces of evidence that we live in an isotropic, homogeneous universe that has been expanding and cooling from an initial state of extreme temperature and density is the cosmic microwave background (CMB). The CMB is an almost perfect blackbody radiation field that permeates space. The best hypothesis for its origin is that the CMB is primordial: it is the radiation field that existed in equilibrium with ionized baryonic matter in the early hot universe. The optically thick universe tightly coupled the radiation and ionized matter through Thomson scattering.

When the expanding universe cooled to close to 3000 K, the ionized matter became neutral and the optical depth of the universe grew dramatically. The decoupled radiation has been free-streaming from this surface of last scattering to current observers with essentially no further interactions with matter. The expanding universe heavily redshifted the radiation field. We observe it today after an expansion factor of close to \( 10^3 \) as a blackbody at a temperature of \( 2.725 \pm 0.002 \)K.

Figure 2.1 shows the CMB emission spectrum measured by the FIRAS instrument on
Figure 2.1. CMB Emission Spectrum: intensity versus frequency. Data are from the FIRAS instrument on the COBE satellite [1].

The COBE satellite [1]. The error bars on the data are smaller than the line thickness in the plot: the agreement with an ideal blackbody spectrum is near perfect. The COBE satellite also confirmed the remarkable isotropy of the CMB. After correcting for the peculiar motion of the Earth with respect to the overall expansion of the universe, deviations from a smooth blackbody spectrum are at 1 part in $10^5$.

### 2.3 Inflationary Expansion

The classical Big-Bang model developed in the previous sections is well supported not only by observations of the CMB, but by close agreement between the observed and predicted chemical abundances produced by Big-Bang nucleosynthesis and the expansion observationally confirmed by the universal Hubble law.
However, there are several problems with this classical model. The two major problems are the so-called flatness problem and the horizon problem.

2.3.1 Flatness Problem

The current observational data show that the universe is very close to geometrically flat (i.e. $\Omega_k \ll 1$ and the sum of the other densities close to unity). Expansion models with a non-zero $\Omega_k$ tend to cause $\Omega_0 = \Omega_m + \Omega_r + \Omega_{\Lambda}$ to diverge rapidly: spherical geometries quickly collapse, making $\Omega_0$ a very large number and hyperbolic geometries quickly expand, making $\Omega_0$ very close to zero. The universe appears finely balanced between these two outcomes, which requires an initial $\Omega_k$ to deviate from zero at 1 part in $10^{60}$. In the classical Big-Bang model, there is no a priori reason for the universe to be flat: it is a troubling coincidence.

2.3.2 Horizon Problem

The particle horizon is defined to be the maximum distance between causally connected events. The particle horizon has been increasing as the universe ages. At the time of the last scattering of radiation and baryonic matter, the particle horizon was much smaller than it is today: causally connected regions at this time subtend an angle of only about 1.12° on the sky [31] for an observer today. Areas on the sky separated by more than about a degree were not in causal contact at this time. These regions were not in thermal equilibrium, according to the classical model, and thus there is no reason for the CMB, on large angular scales, to be as highly isotropic as we observe it today.

2.3.3 Inflation

A solution to the flatness and horizon problem called inflation was proposed in 1981 [32]. Inflation is a rapid superluminal expansion of the very early universe. The hypothesis is that a small region of space, locally flat and in causal contact, exponentially expanded by at least 60 e-foldings to produce the flat isotropic universe we observe today.
There are many models of inflation that produce the required exponential expansion of the early universe. The most common element of most of the models is the presence of a scalar quantum field at extremely high temperatures. If certain conditions about the evolution of this field are met, the field creates a negative pressure, similarly to the negative pressure created by a cosmological constant, and drives an exponential expansion of the universe.

Inflation also provides a natural mechanism for generating small adiabatic density perturbations in the early universe [33]. The deviations from the isotropy that we see today as large scale structure and as primary anisotropies in the CMB are the consequence of these perturbations in the early universe. The current observational promise of the CMB is in quantifying these anisotropies in order to constrain many cosmological parameters and constrain models of inflation and density perturbation production.

2.4 CMB Temperature Anisotropy

The anisotropy of the CMB temperature is often expressed as an angular power spectrum, essentially a spherical harmonic expansion of the temperature on the sky [30]:

\[
\frac{\Delta T}{T}(\theta, \phi) = \frac{T(\theta, \phi) - T_0}{T_0} = \sum_{a,l} a_{l,m} Y_{l,m}(\theta, \phi)
\] (2.8)

The power at different angular scales is most often expressed as a quantity \( C_l \):

\[
C_l = \langle |a_{l,m}|^2 \rangle
\] (2.9)

averaged over the azimuthal component at a given angular scale, \( l \). Figure 2.2 shows the measurements of the CMB temperature anisotropy expressed as an angular power spectrum. The sources of CMB temperature anisotropy can be classified as primary anisotropies, which were imprinted on the CMB at the time of last scattering with baryonic matter, and secondary anisotropies, which were produced between the surface of last scattering and current observers.
2.4.1 Primary Anisotropies

Adiabatic inflationary cosmological models produce density fluctuations on all scales in a very scale invariant way [30]:

$$|\delta_k|^2 = \frac{\delta \rho}{\rho} = A k^n$$  \hspace{1cm} (2.10)

where the density fluctuations are expressed in the Fourier domain in terms of comoving wave number, $k$ ($\delta_k$ is the Fourier component of density fluctuations at scale $k$), $\rho$ is the average density, and $\delta \rho$ is the actual density fluctuation at scale $k$. The size of the density fluctuations determines $A$ and the spectral index, $n$, measures the scale invariance of the initial density perturbations produced during inflation.

These density fluctuations are the principle source of CMB temperature anisotropies.
The evolution of the primordial fluctuations depends heavily on many key cosmological parameters and the nature of this evolution depends on the scale of the fluctuations.

Density perturbations grow with the expansion of the universe due to gravitational collapse on all scales: \( |\delta_k|^2 \propto R^2(t) \). However, smaller angular scales eventually fall into the sound horizon of the tightly-coupled photons and baryonic plasma. Photon pressure can now oppose gravitational collapse and these scales undergo oscillations termed acoustic waves. At the time of recombination, the optical depth for CMB photon scattering quickly shrinks to zero, and the density fluctuations continue their gravity-dominated collapse. We note that dark matter does not participate in the acoustic oscillations and continues collapsing even inside the sound horizon. After recombination, the baryonic matter quickly falls into the dark matter gravitational wells.

The density fluctuations imprint anisotropies onto the CMB at the time of last scattering in several ways. The first is through gravitational redshift: photons must climb out of the gravitational wells produced by collapsing matter after they decouple from this matter producing anisotropies in their temperature through what is called the Sachs-Wolfe effect. On the largest angular scales (for \( l < 100 \)) which did not fall into the sound horizon by last scattering, this is the primary mechanism for producing CMB anisotropies. The large-scale low \( l \) anisotropy is thus a direct probe of the primordial density fluctuation spectrum and has provided solid measurements of \( A \) and \( n \).

The second mechanism for imprinting CMB anisotropies is the compression and rarefaction of the fluctuations undergoing acoustic oscillations. Denser regions emit more photons and appear hotter than more rarefied, cooler regions. Scales that achieved maximum compression at the time of last scattering will produce the most power in the angular power spectrum: this is the source of the acoustic peaks seen in figure 2.2 at \( l > 100 \).

The final source of CMB anisotropy is the velocity of the photon-baryonic fluid at the time of last scattering. Fluctuations undergoing acoustic oscillations will have velocities along an observer’s line of sight which will Doppler shift the CMB photons at the time of last scattering. The Doppler shift will produce the largest anisotropies at velocity extrema,
which occur 90° out of phase with the density extrema. The Doppler shift anisotropies, however, are a much smaller component of the angular power spectrum than the anisotropies produced by the density extrema.

At the smallest angular scales, the primary anisotropies of the CMB are damped. The decrease of optical depth for CMB scattering actually decreased in a small but finite amount of time producing a last scattering layer of finite thickness. The structure in the CMB is suppressed on small angular scales \((l > 1000)\), that is scales that are smaller than the thickness of this last scattering layer.

There is a wealth of information that can be gleaned from measurements of the angular power spectrum of the CMB temperature anisotropy like those shown in 2.2. The CMB angular power spectrum is very sensitive to \(\Omega_0\), the total energy density of the universe (excluding the curvature term), \(\Omega_b\), the baryon density, \(n\), the spectral tilt of the primordial fluctuations and \(\tau\), the optical depth of the post-recombination universe for CMB scattering. Measurements of the largest angular scales of anisotropy are providing a solid probe of both \(n\) and \(\tau\).

The location of the first acoustic peak is extremely sensitive to \(\Omega_0\). The sound horizon at the surface of last scattering acts as a standard "ruler" that depends only on the relatively well constrained redshift of last scattering and the size of the sound horizon. The geometry of the universe will change the angular scale and move the location of the first acoustic peak in the power spectrum. For a flat universe \((\Omega_0 = 1)\), the peak occurs very close to \(l = 220\). For smaller \(\Omega_0\), the geometry of space time is hyperbolic and the peak is pushed to smaller angular scales (larger \(l\)). For larger \(\Omega_0\), the geometry of space time is spherical and the peak is pushed to largest angular scales. Measurements by MAXIMA, DASI, BOOMERANG and the recent WMAP experiment put the peak at \(l = 220\) which is solid evidence for a geometrically flat universe.

The acoustic peaks of the angular power spectrum are also extremely sensitive to the baryonic matter density, \(\Omega_b\). The baryonic fraction determines the effective mass of the photon-baryonic plasma undergoing acoustic oscillations. A higher baryonic fraction en-
hances compression during oscillations, which in turn enhances the size of the odd harmonics of the acoustic peaks.

2.4.2 Secondary Anisotropies

Although CMB photons have been propagating from the surface of last scattering to observers today with relatively minimal interaction with the intervening matter, small secondary anisotropies have been induced. Two major sources of secondary anisotropies exist: interaction with the gravitational potentials of the collapsing matter and interaction with re-ionized matter.

The major secondary temperature anisotropy produced through interaction with gravitational potentials is called the integrated Sachs-Wolfe effect (ISW). In a universe with unchanging gravitational potentials, CMB photons pick up a blue-shift as they fall into a potential and a corresponding and equal red-shift as they climb out of this potential. However, if the gravitational potentials grow or shrink as the photons traverse them, a net red-shift or blue-shift is produced. This effect is most pronounced on larger angular scales ($l < 100$).

Ionized matter can also induce secondary anisotropies through scattering of CMB photons through electrons. The first source of ionized matter is the intergalactic medium. The intergalactic gas is ionized today and the best model for this re-ionization is photoionization of the intergalactic gas during star formation at relatively high redshifts ($z \sim 10$). This scatters a small fraction of CMB photons and reduces the power spectrum by an amount $e^{-\tau}$ on all scales but the largest ones. Here, $\tau$ is the optical depth of the ionized matter from reionization to observers in the current epoch.

The second source of ionized matter is in the hot intracluster gas in galaxy clusters. This gas has heated during the gravitational collapse of the matter forming the galaxy cluster. Gas temperatures can exceed $10^8 K$ depending on the mass of the cluster, and the gas is thus completely ionized. CMB photons passing through the cluster Compton scatter with the hot electrons and their energy is increased in a process called the Sunyaev-Zeldovich
The CMB blackbody spectrum is slightly distorted: there is an excess in the emission spectrum above the peak and a decrement in the emission spectrum below the peak. The magnitude of the SZ effect can be determined through the optical depth of the galaxy cluster [34]:

$$y = \int_{\text{cluster}} \left( \frac{k T_e}{m_e c^2} \right) \sigma_T N_e d\ell$$  \hspace{1cm} (2.11)

where $T_e$ is the intracluster gas temperature, $N_e$ is the number density of electrons, and $\sigma_T$ is the cross-section for scattering between CMB photons and electrons.

Equation 2.11 shows that the Compton scattering optical depth depends directly on the integrated pressure of the intracluster gas. This optical depth allows us to calculate, in the Rayleigh-Jeans limit of the CMB spectrum, the decrement in CMB intensity due to the SZ effect:

$$\frac{\Delta I_\nu}{I_\nu} = -2y$$  \hspace{1cm} (2.12)

Since the SZ decrement is just a fractional change in the emission spectrum of the CMB, it is independent of the redshift of the galaxy cluster in which scattering occurs. The SZ effect contributes to the CMB temperature anisotropies on scales above $l = 2000$.

This redshift independence gives the SZ effect its true power: SZ galaxy cluster surveys will effectively find galaxy clusters down to a cluster mass limit (that depends on observational sensitivity) at all redshifts. The formation of massive clusters is very sensitive to structure formation and several key cosmological parameters such as $\Omega_m$, $\Omega_\Lambda$, and $w$ which parameterizes the equation of state of dark energy [35]. The complete cluster surveys that can be obtained using the SZ effect serve as a powerful probe of cosmology and will allow tight constraints to be placed on these parameters. Figure 2.3 shows simulated cluster number density versus redshift for a fiducial cosmology along with the expected number densities for different sets of cosmological parameters.

In addition, a combination of X-ray observations and SZ observations of the same cluster provides a good estimate of the size of the cluster: in this way galaxy clusters can be used
as standard rulers in much the same way as type 1a supernovae, allowing us to measure the Hubble parameter, $H_0$, and the expansion history of the universe to moderately high redshifts ($z < 2$) [36].

### 2.5 CMB Polarization Anisotropy

The standard adiabatic inflationary Big-Bang cosmological model predicts CMB polarization anisotropy. In fact every cosmological model with fluctuations at the surface of last scattering predicts CMB polarization anisotropy. The reason is that the cross-section for Thompson scattering, the dominant interaction between matter and radiation, is polarization dependent:

$$\frac{d\sigma_T}{d\Omega} \propto |\hat{\epsilon} \cdot \hat{\epsilon}'|^2$$

(2.13)

where $\hat{\epsilon}$ is the incident polarization vector and $\hat{\epsilon}'$ is the scattered polarization vector.
A local net quadrupole moment in photon temperature at the surface of last scattering on scales larger than the thickness of the surface of last scattering produces a linear polarization [37].

Net quadrupole moments on all scales arise from fluctuations hypothesized to have been created during the inflationary expansion of the universe: both scalar (density fluctuations) and tensor (long-wavelength gravitational waves) perturbations produce quadrupole moments and thus imprint themselves as CMB polarization anisotropy.

A linear polarization can be described in terms of two of the Stokes parameters \[ Q = \langle a_x^2 \rangle - \langle a_y^2 \rangle \]
\[ U = \langle 2a_x a_y \cos \theta_x - \theta_y \rangle \]

where \( a_x \) is the amplitude of the horizontal component of the electric field vector and \( a_y \) is the amplitude of the vertical component. \( Q \) measures the tendency of the polarization to be horizontal (positive \( Q \)) or vertical (negative \( Q \)) and \( U \) measures the tendency of the polarization to be at \( \pm 45^\circ \). Unpolarized light has \( Q = U = 0 \). The Stokes parameters have the disadvantage of being coordinate dependent: \( Q \) and \( U \) mix under a coordinate rotation.

The typical way, then, of describing a CMB polarization field is with coordinate-independent E- and B- components. The use of E and B polarization "modes" not only has the advantage of coordinate independence, E and B modes also transform differently under a parity operation: E modes are symmetric and B modes are antisymmetric. Thus the CMB polarization field can be effectively cleaved into two separate components: we will see that slightly different classes of fluctuation work to produce these two components at the surface of last scattering.

Scalar perturbations at the surface of last scattering produce E-mode polarization anisotropy that is complementary to CMB temperature anisotropy. At intermediate angular scales the polarization anisotropy is most pronounced when the velocity of the acoustic oscillations is at a maximum (producing the largest local quadrupole at this scale). The acoustic peaks we see in the CMB temperature anisotropy power spectrum predict a series
of similar peaks in the CMB polarization anisotropy power spectrum but interleaved (90° out of phase) with the temperature peaks.

Tensor perturbations, induced by long-wavelength gravitational waves produced during inflationary expansion, polarize the CMB with both E-mode and B-mode polarization signals. This is the only known mechanism that produces B-mode polarization at the surface of last scattering. Detection of B-mode polarization is therefore a direct measurement of one of the consequences of an inflationary expansion of the universe and is therefore a key piece of experimental evidence for the adiabatic inflationary Big Bang model.

On small scales ($l > 400$), gravitational lensing by intervening matter between the surface of last scattering and current observers can convert E-mode polarization into B-mode polarization [38]. This “contamination” of the pure tensor-produced B-mode signal at smaller scales will limit the ability to precisely measure the contribution from primordial tensor fluctuations [39]. However, the B-mode signal from gravitational lensing contains information about early structure formation and a characterization of the B-mode signal will put a tighter constraint on several cosmological parameters.

Measurements of the CMB polarization anisotropy provide another snapshot of the dynamics of the universe at the time of last scattering and are very complementary to measurements of CMB temperature anisotropy. In particular measurements of the E-mode CMB polarization anisotropy will allow us to directly confirm whether the density fluctuations that give rise to the temperature anisotropy are purely the adiabatic fluctuations expected from inflationary expansion. Large angular scale E-mode polarization breaks the degeneracy between optical depth to reionization $\tau$ and the spectral tilt of the primordial density fluctuations, $n$ that exists in analyses of only the temperature anisotropy.

Finally, detection of B-mode polarization anisotropy on intermediate angular scales ($l = 100$), below the E-mode contamination from gravitational lensing, is a powerful and key probe of inflation. The power in B-mode anisotropy at this scale is will probe the amplitude of the tensor fluctuations at the surface of last scattering. This amplitude in turn is very dependent on the energy at which inflation occurs. Constraints on the level of B-mode
polarization will allow us to accept or rule out many classes of inflationary model and a detection of B-mode polarization will put an energy scale on the physics of inflation. Figure 2.4 shows a space of $r$ (tensor to scalar primordial fluctuation ratio) versus $n$ (primordial spectral tilt) along with the parameter space occupied by several classes of inflationary models. Constraints on B-mode polarization allow us to place direct constraints on this parameter space and thus inflationary models.

Figure 2.4. $r$ (tensor to scalar ratio) versus $n$ (primordial spectral tilt). Blue: $V \propto e^{\phi/\mu}$; Red: $V \propto 1 - (\phi/\mu)^2$; Black: $V \propto \phi$ from [4]
2.6 Motivation for Bolometer Arrays

Measurements of the CMB temperature anisotropy and preliminary detections of CMB polarization anisotropy have yielded immense amounts of information about the structure, composition, and dynamics of our universe. The adiabatic inflationary Big-Bang model has been confirmed as an effective and viable model for our universe. There is still enormous amounts of information to gain through ultra-sensitive measurements of the CMB.

As outlined in the previous section galaxy cluster surveys utilizing the SZ effect will map out structure formation at intermediate redshifts and place strong constraints on several cosmological parameters. Effective mass-limited surveys will require sensitivities to several $\mu K$ on angular scales of around 1'. However, to efficiently map out large areas of the sky during galaxy cluster searches to these sensitivity levels, extremely sensitive receivers are needed. To achieve the required sensitivities, large arrays of bolometers will be used [3].

Full characterization of the CMB polarization anisotropy power spectrum will be a major experimental challenge. Figure 2.5 shows the expected levels for the E-mode and B-mode polarization signals. Adiabatic inflationary cosmological models predict that the power in the E-mode angular power spectrum is at least an order of magnitude smaller than the power in the CMB temperature anisotropy. The angular power spectrum of B-mode polarization is at least another order of magnitude smaller under the most optimistic assumptions about the energy scale of inflation. A major increase in experimental sensitivity, which can only be achieved through the deployment of large-scale bolometer arrays, is needed to extract the promising and exciting science from the polarization of the CMB.
Figure 2.5. Expected E-mode and B-mode polarization spectra. A range of inflationary energy scales is considered in predicting the B-mode signal.
Chapter 3

Sensors

In this chapter we discuss the transition-edge sensor bolometer. We motivate the choice of bolometers in general for cosmic microwave background observation and describe the choice of transition-edge sensors in particular. We summarize the history of transition-edge sensor development and describe in detail the operation of a transition-edge sensor.

3.1 Measuring Microwave Radiation

There are two classes of detectors used to observe mm-wave radiation: coherent detectors and direct detectors. In coherent detection, mm-wave radiation is coherently received with an antenna and amplified with a HEMT amplifier (high electron mobility transistor) [40]. After amplification the signal is measured with a diode, or more typically, down-converted in frequency with a very stable local oscillator and amplified with a lower bandwidth IF amplifier and then measured with a diode. Coherent detection preserves the phase information of the mm-wave radiation and has typically provided very close to quantum limited sensitivities below 100 GHz. Quantum noise, however, severely degrades the performance of coherent detection above 100 GHz.

Observations of the cosmic microwave background at 150 GHz are attractive because the joint contributions from both lower frequency foregrounds (galactic synchrotron radiation)
and higher frequency foregrounds (dust emission) is minimized compared to the spectral intensity of the CMB. Direct detection, also called square law detection, incoherently measures the power of the radiation: the output of a direct detector is proportional to the power of the input signal, or the square of the input signal amplitude.

### 3.2 Bolometers

The direct detectors commonly used to measure the cosmic microwave background are bolometers. Bolometers are thermal detectors of radiation. They consist of an absorbing element with a heat capacity $C$ which is thermally connected to a heat sink at $T_s$ through a thermal conductance $G$. When the bolometer absorbs a radiation power $P$ from the sky, its temperature increases to

$$T_b = T_s + \frac{P}{G} \left(1 - e^{-t/\tau}\right)$$

where $\tau = C/G$ is the intrinsic thermal time constant of the bolometer [41].

#### 3.2.1 Absorbing Element

Absorbers typically consist of a thin layer of metal deposited on a suspended or isolated backing structure. The bolometers used in the multiplexer development have absorbers that use silicon nitride as a backing structure. A 1 $\mu$m layer of Si$_3$N$_4$ is deposited using low pressure chemical vapor deposition on a silicon wafer. The absorber material and structure is then deposited on top of this backing layer.

We have multiplexed bolometers with two different kinds of absorber. The first, antenna-coupled bolometers, couple mm-wave radiation received by a lithographed antenna into a matched 50 $\mu$m long titanium load resistor that is deposited on the Si$_3$N$_4$ layer. A dry gas etch removes the silicon from behind the silicon nitride and produces a released suspended structure thermally decoupled from the rest of the wafer. As mm-wave radiation is deposited in the load resistor, it heats up the suspension.
The second type of multiplexed bolometer, direct absorber coupled bolometers, consists of a large spider-web of gold deposited on the $\text{Si}_3\text{N}_4$ layer. The spider-web geometry spacing is small enough to effectively absorb mm-wave radiation while simultaneously providing a very small cross-section for absorption of cosmic rays. The thickness of the gold layer is designed so that the impedance of the gold mesh closely matches the impedance of free space: $377 \, \Omega/\Box$.

### 3.2.2 Thermistor

The rise in bolometer temperature due to absorbed mm-wave radiation is measured with a thermistor. In the traditional state of the art bolometer technology used in experiments such as MAXIMA and BOOMERANG, composite bolometers consisting of spider-web absorbers and semiconducting thermistors were used. The thermistors are individually attached to the spider-web absorbers by gluing or bump-bonding. The thermistors typically consist of a material called NTD-Ge: neutron transmutation doped germanium. The resistance of these bolometers is strongly dependent on temperature [42]:

\[
R_{NTD} = R_0 e^{\Delta/T^{1/2}} \tag{3.2}
\]

The temperature of such a composite bolometer is typically read out by applying a current bias to the NTD-Ge thermistor and reading out the voltage changes.

Such a composite bolometer has several disadvantages in scaling the technology to large arrays of detectors. The first is that the thermistors have to be individually added by hand to the photolithographically produced absorbers. Scaling this procedure up to thousand-pixel arrays will be extremely onerous and time-consuming. Perhaps more importantly is the readout disadvantage: the readout of the high-impedance thermistors (the sensors typically operate at $R_{NTD} \sim 3M\Omega$) is performed with JFET transimpedance amplifiers. Possessing a multiplexed version of the readout is essential in the instrumentation of large arrays. The JFET transimpedance amplifier is limited in both bandwidth and sensitivity is not easily extended to a multiplexed readout.
A competing thermistor technology for mm-wave bolometers is the superconducting transition-edge sensor (TES) \[43\]. A TES bolometer can be made photolithographically, with the deposition of the Si$_3$N$_4$, absorber, and thermistor merely different individual lithography steps in a multi-step process. The production of a single pixel can be extended to produce large arrays of pixels. Since the bolometers are produced identically, we expect and routinely observe very uniform operating parameters in the detectors across an array. Finally, and perhaps most significantly, clear multiplexed readout strategies for TES bolometers exist and have been under extensive development for the last five years \[44\].

### 3.3 Transition-Edge Sensor Bolometers

A transition-edge sensor consists of a thin film of superconducting material that is electrically biased in the transition between its normal state and its superconducting state. The sharp transition in electrical resistance produces a very sensitive thermistor: very small changes in temperature produce large changes in resistance. The width or sharpness of the transition is characterized by the dimensionless parameter $\alpha$:

$$\alpha = \frac{(T/R)dR}{dT} = \frac{d(\log R)}{d(\log T)} \quad (3.3)$$

TES bolometers have $\alpha$ anywhere from 50 to 1000. The film used as a thermistor for the multiplexed bolometers consists of a bilayer of aluminum and titanium: 400 Å of aluminum underneath 400 Å of titanium. The transition temperature of aluminum, $T_c = 1.2$ K, is too high to use as a thermistor material. We typically cool focal plans for mm-wave astronomy with $^3$He sorption fridges which can achieve temperatures of 200 to 300 mK. We would like to operate our TES bolometers at a temperature slightly higher than this. Thus, the thermistors should then, by design, have a $T_c$ between 450 and 550 mK.

Adding a layer of titanium, which has a transition temperature $T_c = 0.39$ K, lowers the transition-temperature of the aluminum through the proximity effect.

The proximity effect is described in detail in \[45\] and \[46\]. At a temperature below the transition temperature of pure aluminum, cooper pairs from the superconducting aluminum
layer leak into the normal titanium layer and quasiparticles (electrons) leak from the normal titanium layer into the superconducting aluminum layer. The net effect is that the close physical proximity of the normal titanium suppresses or reduces the transition temperature of the aluminum. The amount of suppression depends on the thicknesses of the aluminum and titanium layers and the pure transition temperatures of these materials.

Figure 3.1. Data showing the proximity effect in a copper/lead sandwich. The normalized transition temperature of lead is plotted against the lead thickness for several thicknesses of copper (from [5])

Figure 3.1 illustrates this effect with a bilayer of lead (superconducting layer) and copper (normal layer). The presence of the copper universally suppresses the $T_c$ of the lead, but the effect depends strongly on the thickness of the lead and the copper layer. The proximity effect is the strongest when the layer thicknesses are very similar to the coherence length of the cooper pairs in the materials. In practice, with a 400 Å layer of aluminum and a 400 Å of titanium, we achieve bilayer transition temperatures of 500 mK.
3.3.1 Operation

A TES bolometer is operated by applying a voltage bias to the film. Changes in the TES temperature produce changes in the operating resistance of the film and thus change the current flowing through the film. The positive sign and relatively high value of $\alpha$ for superconducting films gives the TES a very attractive property: strong electrothermal feedback [43].

When the TES is operating in its transition, small increases in optical power from the sky tend to drive the TES towards the normal part of its transition and thus increase its resistance. The electrical bias power of the TES,

$$P_e = \frac{V^2_b}{R}$$

(3.4)

decreases because of this resistance change (i.e. $\frac{dP_e}{dT} < 0$). This negative feedback tends to keep the total power deposited into the superconducting film constant:

$$P_T = P_{\text{optical}} + P_e \approx \text{constant}$$

(3.5)

We quantify the dynamic range, responsivity, sensitivity, and bandwidth of a TES bolometer by following [25]. We start by conserving energy in the bolometer in the presence of optical power, electrical power, and a varying optical signal $\delta P e^{i\omega t}$:

$$P_{\text{optical}} + \frac{V^2_b}{R} - \frac{V^2_b}{R^2} \frac{dR}{dT} \delta T e^{i\omega t} = \overline{G}(T_b - T_s) + (G + i\omega C)\delta T e^{i\omega t}$$

(3.6)

where $\overline{G}$ is the average thermal conductance between the TES temperature, $T_b$ and the bath temperature, $T_s$.

Equation 3.6 can be split into two equations, one that gathers terms that are constant in time and other that gathers terms that vary in time. The constant terms express the steady state conservation of energy:
The sum of the average optical power and electrical power must match the power leaving the bolometer through the weak thermal link to the bath.

The time varying terms form:

$$\delta P e^{i\omega t} = \left( \frac{P e^{-\alpha}}{T} + G + i\omega C \right) \delta T e^{i\omega t}$$  \hspace{1cm} (3.8)

where $\alpha$ has been defined in equation 3.3. The terms in the brackets of equation 3.8 have the units of thermal conductance and define the effective complex thermal conductance of a TES bolometer under strong electrothermal feedback [25]:

$$G_{eff} = \frac{P e^{-\alpha}}{T} + G + i\omega C$$  \hspace{1cm} (3.9)

Equation 3.9 shows that this feedback effectively increases the bolometer $G$. This is equivalent to saying that the feedback "nulls" temperature changes and for strong feedback, the TES remains at a fixed temperature and thus position in the transition for relatively large changes in incident power.

The strength of the electrothermal feedback can be defined as the ratio of the change in electrical bias power to a change in TES power, a quantity completely analogous to loop gain in a pure electrical circuit:

$$L(\omega) = \frac{-\delta P_e}{\delta P_{total}} = \frac{P e^{-\alpha}}{GT_b(1 + i\omega \tau_0)} = \frac{L}{1 + i\omega \tau_0}$$  \hspace{1cm} (3.10)

where $L = \frac{P e^{-\alpha}}{GT_b}$ is the loop gain of the system; $L$ exhibits a single-pole decrease with time constant $\tau_0 = C/G$. Temperature changes induced in the superconducting film under feedback are decreased by the factor of the loop gain compared to the change that would be induced without feedback.

We are interested in measuring a electrical change in the bolometer with the incident
optical power $\delta P$. Since we voltage bias our bolometers, we are thus interested in the change in current for a given change in incident optical power. This is called the responsivity $S_i$ of the bolometer:

$$S_i = \frac{\delta I}{\delta P} = \frac{-1}{V_b} \frac{L}{L + 1} \frac{1}{1 + i\omega\tau}$$

(3.11)

where $\tau = \tau_0/(L + 1)$ is the effective thermal time constant of the TES bolometer (intrinsic time constant decreased by the loop gain).

By connecting a TES bolometer to a high-performance ammeter, we can measure directly the changes in optical power incident on the detector. Equation 3.11 shows an important and fundamental property of superconducting detectors under strong electrothermal feedback: for large loop gains $L \gg 1$, the responsivity for slow optical signals is a constant value independent of the applied bias power: $S_i = \frac{-1}{V_b}$. The electrothermal feedback thus linearizes the response of the TES and produces a very stable and linear responsivity.

Although the responsivity expressed in equation 3.11 is valid for small incident powers, that is incident powers that produce small changes in operating temperature ($\delta T \ll T_b$), there is an inherent limitation to the dynamic range of a TES bolometer. If the incident power exceeds the right hand side of equation 3.7, $G(T_b - T_s)$, the thermistor is driven into what is called “saturation”, i.e. driven completely normal. When the incident power saturates the bolometer, the responsivity rapidly drops from the ideal $-1/V_b$ to zero. Thus, TES bolometers must be designed by carefully estimating the maximum anticipated incident power and ensuring that the thermal conductance, $G$, operating temperature $T_b$, and the bath temperature, $T_s$, result in enough power margin to avoid saturation.

### 3.3.2 Dark Sensitivity

In addition to the responsivity, we are interested in the sensitivity of a TES bolometer. Sensitivities are often quoted as Noise Equivalent Power (NEP), which is defined to be the amount of power incident on a TES that produces a signal that equals the noise in a $1 \text{ Hz}$ bandwidth (or, equivalently, the amount of power needed for a signal achieve a signal to
noise ratio of 1 in after 1/2 seconds of integration). There are several sources that contribute to the NEP of a TES bolometer. If we consider a dark bolometer only (no incident optical power), have

\[
\text{NEP}^2 = \gamma 4kT^2G + \frac{4kT/R}{S_i^2}(\frac{\tau}{\tau_0})_2 \left(1 + \omega^2\tau_0^2\right) + \frac{i_{\text{readout}}^2}{S_i^2}
\]  

(3.12)

The first term in equation 3.12 arises from thermal fluctuations in the conductance connecting the the bolometer absorber to the heat bath. The spectral density of these fluctuations is \(4kT^2G\) in the limit of vanishing temperature gradients in the conductance itself. For the more realistic case of an approximately 200 mK gradient across the conductance, this formula is adjusted by a factor \(\gamma\) which is of order unity [47]. The second term in equation 3.12 arises from the fact that the thermistor is resistive and therefore produces Johnson current noise with a spectral density of \(4kT/R\). However, these fluctuations are reduced in the presence of strong electrothermal feedback by approximately a factor of the loop gain.

Any practical ammeter measuring the current flowing through the TES will introduce current noise, \(i_{\text{readout}}\) and degrade the bolometer sensitivity. This current noise, divided by the responsivity, contributes another term to the bolometer NEP. The second and third term should be much smaller than the first in order for the NEP of a dark bolometer to be dominated by the thermal fluctuation noise. In particular, TES bolometers readout with superconducting quantum interference devices (SQUIDs) (discussed in chapter 5) come very close to being limited by thermal fluctuation noise.

### 3.3.3 Photon Noise

When there is indicent power on a TES bolometer, another term contributes to its NEP: photon noise. Since optical power is carried with photons, the Poisson statistics of photon arrival affect the sensitivity with which we can measure this power. In the case of pure Poisson statistics (ignoring any photon correlation associated with “bunching”), the RMS fluctuation in the number of photons is:
\[ \langle N^2 \rangle = N = P_{\text{optical}}/h\nu \quad (3.13) \]

This can be expressed as a fluctuation in power:

\[ \langle P^2 \rangle = N(h\nu)^2 \quad (3.14) \]

and for a 1s integration, an NEP of:

\[ \text{NEP}_{\text{photon}} = \sqrt{\frac{2P_{\text{optical}}hc}{\lambda}} \quad (3.15) \]

\[3.4 \text{ TES Design} \]

The formulas derived in the preceding section for TES dynamic range, sensitivity, and bandwidth provide the design guidelines for a TES bolometer. The first decision is the choice of an operating temperature. For photon-limited performance we require that

\[ \gamma 4kT^2G < \frac{P_{\text{optical}}hc}{\lambda} \quad (3.16) \]

For mm-wave radiation and background black-body loading in the 10s of kelvin, operating temperatures of < 1 K will produce photon-limited NEPs. The ³He sorption refrigerator is a mature and robust refrigeration technology that can provide cooling power of around 50 – 80 µW at \( T_s \sim 250 \) mK. To minimize \( \gamma \), the detector operating temperature is usually chosen to be a factor of 1.5 higher than the bath temperature. Thus, operating temperatures of \( T_c \sim 400 – 500 \) mK, routinely achieved with aluminum and titanium bilayers, are an ideal choice.

Once \( T_b \) and \( T_s \) are chosen to insure photon-noise limited performance, the detector \( \overline{G} \) must be chosen to satisfy:

\[ P_{\text{max}} < \overline{G}(T_b - T_s) \quad (3.17) \]

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The maximum anticipated incident power, $P_{\text{max}}$, depends on several factors: the maximum temperature of the radiation source (typically a 77K black body), the throughput of the optics that couple the radiation source to the detector, and the bandwidth of the radiation. A blackbody source has a spectral intensity:

$$B(\nu, T)d\nu = \frac{2\nu^3 d\nu}{e^{h\nu/kT} - 1}$$

(3.18)

If we assume our brightest source of radiation is a blackbody at a temperature $T$, then to determine the maximum optical signal at our detector, we multiply equation 3.18 by the throughput of the coupling optics, $A\Omega$, and integrate over the bandwidth defined by the filters in our system. For example, a 77K blackbody, at $\lambda = 2$ mm and with a 20% bandwidth, produces an optical power of 60 pW. This places a lower limit on $G$ of $\sim 220$ pW/K.

Finally, the bandwidth of a TES bolometer determines the speed at which the detector can respond to incident radiation. However, for TES bolometers with strong electrothermal feedback, the intrinsic time constant $\tau_0 = C/G$ is decreased by a large factor and the detector response to incident radiation is in practice limited by the time it takes this incident power to thermalize in the absorber [48]. We will see in much more detail in chapter 6 that the increase in detector bandwidth due to electrothermal feedback is detrimental in multiplexing their readout and that the intrinsic time constant of the detectors needs to be increased by adding a large additional heat capacity, $C$, to the absorbers.

### 3.5 Readout

Since the TES bolometer is voltage biased, a sensitive ammeter is used to measure the change in TES current in response to changing incident radiation. There are several requirements for the readout ammeter:

- low input impedance $Z_{\text{in}}$
- low input current noise $i_{\text{readout}}$
large forward gain

The first requirement ensures that the voltage bias of the TES bolometer remains a pure voltage bias in the presence of the readout ammeter. This is equivalent to requiring that the presence of the readout ammeter not effect the current flowing through the thermistor. The TES bolometers used to develop the readout multiplexer typically have normal resistances of close to 1Ω, requiring that \( Z_{in} \ll 1\Omega \).

The second requirement ensures that the readout of the TES bolometer does not degrade the NEP of the detector. For the example bolometer in the previous section, with \( G \sim 220 \text{ pW/K} \), the expected thermal fluctuation noise produces a current noise of \( \sim 10 \text{ pA} \sqrt{\text{Hz}} \). Thus, we require that \( i_{\text{readout}} \ll 10 \text{ pA} \sqrt{\text{Hz}} \).

Finally, the third requirement ensures that the bolometer current noise be amplified to a level well above the input voltage noise of subsequent room temperature amplification stages in the readout.

In practice, dc-SQUID ammeters are used to readout the current from voltage-biased TES bolometers. SQUID ammeters easily satisfy the requirements of low input impedance and input current noise and large forward gain. In addition, they have the large bandwidth required to extend single channel readout to multiplexed readout. The principles, operation, and practical use of dc-SQUIDS for multiplexed readout of TES bolometers will be discussed in detail in chapter 5.
Chapter 4

Readout Multiplexing

In this chapter, we discuss the two major, complementary readout multiplexing architectures: time-domain multiplexing (TDM) and frequency-domain multiplexing (FDM). We will outline the advantages to each scheme and motivate the development of FDM.

4.1 Overview

Readout multiplexing is essential for the feasible deployment of large transition-edge sensor bolometer arrays. One of the reasons CCD technology plays a dominant role in visible and infrared radiation receivers is the ease and maturity of readout multiplexing of CCD camera pixels. This has, in part, led to the affordable several mega-pixel commercial digital cameras available today as well as the mega-pixel infrared cameras being designed for the SNAP satellite [49].

There are three strong drivers to developing multiplexed readout for TES bolometers. As discussed in the previous section, TES bolometers are typically cooled to below 1 K. Individual wiring to each sensor, in the case of non-multiplexed single-channel readout, makes cooling an array of more than a few tens of pixels cryogenically infeasible. As discussed in both the previous and the next section, dc SQUIDs (superconducting quantum interference devices) are used as the first stage of TES bolometer readout. These components are
very costly and become a significant term in the total cost of building a mm-wave receiver. Readout multiplexing reduces the number of SQUID amplifiers needed, lowering the cost of building and integrating a receiver. Finally, the sheer number of wires that connect from 300K to 1K, in terms of wiring complexity and cost, grows rapidly with large arrays under single-channel readout. Reducing this wiring complexity is essential in instrumenting large arrays.

Figure 4.1. Schematic comparison of MUX technologies: a) TDM and b) FDM
4.2 TDM versus FDM

4.2.1 Basic Concepts

Readout multiplexing combines signals from many sensors \(N\) before a cryogenic amplification stage. The signals are brought out to 300 K on \(1/N\)th the number of readout lines and then separated again with electronics at 300 K.

Two multiplexing (MUX) technologies are currently being pursued: time-domain multiplexing (TDM) and frequency-domain multiplexing (FDM). Figure 4.1 shows a very rough schematic of the two techniques: in TDM, a single cold readout amplifier is connected to \(N\) sensors through individual cryogenic switches. The readout amplifier sequentially switches through all \(N\) sensors, spending \(1/N\)th of its time at each sensor. In FDM, each sensor is simultaneously readout with a single cold readout amplifier. However, each of the sensors is biased at a unique sinusoidal frequency separating the sensor signals in frequency space.

The TDM and FDM multiplexing techniques are complementary. The overall concept is identical for both techniques: a set of \(N\) bolometer signals is multiplied by a set of \(N\) orthogonal functions (square waveforms in the case of TDM, sinusoidal waveforms in the case of FDM). The signals are summed, allowing multiplexing, then separated again by another multiplication with the same set of orthogonal functions [50].

4.2.2 Circuit Schematics

TDM Readout multiplexing of TES detectors was first proposed by the cryogenic detector group at the National Institute for Standards and Technology (NIST) [44]. Several generations of TDM MUX were explored. The MUX technology has matured and is in use on several receivers employing large arrays of TES detectors [27] [6]. Figure 4.2 shows the basic TDM MUX circuit. Each TES detector in a given MUXed column is connected in series to a single SQUID which acts as a cryogenic switch (a large cold inductor is also placed in series with the TES detector to bandwidth limit the Nyquist noise from the detector). Address lines exist that allow only one of the SQUID switches to be biased (ON) at a time.
The outputs of the $N$ SQUID switches are inductively coupled to a summing loop which sums the signals and in turn couples them into a second-stage readout SQUID. The output of this SQUID drives a third amplification stage before connection to 300K electronics. The number of address lines is reduced further by connecting a row of $M$ SQUID switches to a single address line: all of these $M$ SQUID switches are active simultaneously, allowing sequential individual readout of each of the $N$ members of the column.

FDM readout multiplexing was first proposed at UC Berkeley [51]. A second generation of FDM MUX has been demonstrated and the maturing technology will be in use as the
readout for several mm-wave receivers [7]. Two other groups, ISAS (Japan) [28] and SRON/VTT (Finland, The Netherlands) [29] are now pursuing parallel developments of FDM MUX technology. Figure 4.3 shows the basic FDM MUX circuit as developed at UC Berkeley. Each detector in a set of $N$ is biased with a sinusoidal bias at a unique frequency $f_n$ (frequencies typically range from 300kHz to 1MHz). An tuned filter consisting of cold inductor and cold capacitor is placed in series with each TES bolometer. The tuned filter has two functions: it bandwidth limits the broadband Nyquist current noise from the resistive TES bolometer (without the filter, the sensitivity of MUXed detectors rapidly degrades as the number of MUXed detectors increases). In addition, it allows the $N$ detectors to be biased with a single pair of wires: a ”comb” of $N$ bias frequencies is sent down this single pair. The tuned filter passes the appropriate bias carrier and suppresses the other $N-1$. The $N$ sensors with their series filters are connected in parallel and the currents are summed and readout with a single SQUID readout amplifier. Warm demodulation electronics separate and recover the individual detector signals.
4.2.3 Comparison

Although TDM and FDM are in principle complementary readout multiplexing technologies, differences arise in real implementations of these systems [52]. Several important properties, and how they compare between TDM and FDM, are discussed in detail below.

**Multiplexed Readout Bandwidth**

The multiplexed readout bandwidth determines how many detectors can be multiplexed. For a TDM system, the readout bandwidth is determined by the open-loop bandwidth of the readout amplifier (SQUID amplifier). This bandwidth determines the rate of switching between multiplexed detectors. Current achieved open-loop bandwidths are on the order of 1-2 MHz and is limited by the response time in the circuitry that samples the SQUID outputs and toggles the SQUID switches.

For a FDM system, since all the detectors are simultaneously sampled, the readout bandwidth is determined by the closed-loop bandwidth of the readout amplifier (SQUID amplifier). The closed loop bandwidth determines the number of detectors that can be accommodated in a FDM system. Current achieved closed-loop bandwidths are also close to 1-2 MHz. The feedback loop operating the SQUID readout amplifier contains warm components. Wire lengths between the components at 300K and at 4K currently limit the closed-loop bandwidth: round-trip wire lengths of 0.2 to 0.3m (lengths shorter than this are not cryogenically feasible) correspond to maximum closed-loop bandwidths of 1-2 MHz. Chapter 8 will discuss moving the entire feedback loop to 4K to increase the closed-loop bandwidth by a large factor.

Of course, the number of detectors that can be multiplexed also depends on the detector bandwidth. Faster detectors (larger bandwidth) will use more of the multiplexed readout bandwidth and fewer detectors can be multiplexed compared to slower detectors (smaller bandwidth).
Multiplexed Readout Noise

For TDM and FDM systems, the readout noise comes from two sources: the broadband Nyquist noise from the TES detector itself and the readout SQUID amplifier noise.

For the TDM system, a low pass L/R noise aliasing filter is used (the R is typically the TES detector operating resistance; the L is the combination of the SQUID input inductance and an additional anti-aliasing inductor placed in series with the TES detector). This filter bandwidth limits the Nyquist noise from the detector and effectively avoids this noise from aliasing down and degrading the TES detector performance. However, the readout SQUID amplifier noise is broadband. For the NIST TDM system the noise contribution from the readout SQUID is small. However, as the readout bandwidth increases and the number of sensors being multiplexed correspondingly increases, the contribution from the readout SQUID amplifier grows.

The FDM system bandwidth limits the TES detector Nyquist noise with a cold tuned filter in series with the detector. In this case, noise-aliasing is not a concern since the readout SQUID amplifier is constantly sampling all the detectors. Instead the filter is designed to prevent the Nyquist noise contribution from the other $N - 1$ sensors from degrading the multiplexed TES detector performance. The constant sampling of all the detectors in the FDM system limits the contribution from the readout SQUID amplifier: only the small bandwidth about the detector bias frequency contributes to the readout noise.

Power Dissipation

At the coldest temperature in the system (the detector temperature, $T < 1K$), the TDM system dissipates a small amount of power due to the active SQUID switching. This has been reduced to $\sim 130$ nW for a 1000 channel system and can probably shrink further. Since all components at the detector temperature are passive in the FDM system, there is zero power dissipation at this stage.

At the readout SQUID stage, typically at a temperature near 4K, the power dissipation is from the readout SQUID biases. Both systems have a comparable amount of dissipation.
Table 4.1. Comparison of TDM and FDM system properties

<table>
<thead>
<tr>
<th>Property</th>
<th>TDM</th>
<th>FDM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Readout Bandwidth</td>
<td>1-2 MHz</td>
<td>1-2 MHz</td>
</tr>
<tr>
<td>Noise</td>
<td>increases with $\sqrt{n}$</td>
<td>independent of $n$</td>
</tr>
<tr>
<td>Power (&lt; 1K)</td>
<td>100 nW</td>
<td>0</td>
</tr>
<tr>
<td>Power (300 K)</td>
<td>300 W</td>
<td>3 kW</td>
</tr>
<tr>
<td>Filters</td>
<td>L/R</td>
<td>LC</td>
</tr>
</tbody>
</table>

depending on the number of channels and the MUX factor: a few $\mu$W of power is dissipated at this temperature for a 1000 element array.

The systems currently differ markedly in the amount of power dissipated at 300K. The TDM system has benefited from several more years of development and the 300K power dissipation for a 1000 channel experiment is close to 300W. The FDM system is about an order of magnitude larger. Much of this power expenditure comes from the current generation of sinusoidal bias carrier generation integrated circuits (ICs). A switch to low power, all digital bias carrier generation is planned. This will reduce the FDM system power dissipation at 300K by close to an order of magnitude. A power dissipation of 300W more than meets the requirements for flight on a long-duration balloon or space-borne instrument.

4.3 FDM Development

The readout multiplexing development at Berkeley has focused on an FDM system. There are several reasons for this choice. The most important is the fact that the TDM system requires one SQUID for every detector to act as the cryogenic switch in addition to the readout SQUID amplifiers. Implementing a TDM system requires fabrication of hundreds to thousands of SQUID switches and integration of these switching SQUIDS with a TES bolometer array. Our experience with building TES detectors is solid but our group does not have a history of SQUID fabrication. The choice was made to simplify the multiplexer development by pursuing the FDM system and purchasing the readout SQUID amplifiers from an outside vendor.

There are several other advantages to the FDM system, most of which are outlined
above, that make it an attractive technology to pursue. Zero power dissipation at the coldest stages is a real advantage when considering the extremely limited cooling capacities available in a long-duration balloon flight or satellite.

The sinusoidal bias of the detectors in the FDM system is at a much higher frequency than any possible mechanical resonance or vibration in the receiver, telescope, balloon, and satellite that houses the readout system. This makes FDM systems, in principle, extremely insensitive to mechanical pickup or vibrational heating of the sensors. Chapter 7 discusses a set of measurements that confirms this insensitivity to vibrational heating. Operating the sensors with a sinusoidal bias also avoids the ubiquitous low frequency gain variations that can plague many systems. For instance, SQUID arrays typically have a $1/f$ knee near 50 to 100 Hz. An FDM system effectively avoids this gain variation.
Chapter 5

SQUIDs

In chapters 3 and 4, SQUID amplifiers were briefly described as the preferred method of measuring the current flowing through a TES detector. The combination of extremely low input impedance, high sensitivity (low noise temperature), and high forward gain make SQUID amplifiers almost ideally suited to readout TES detectors. However, significant improvements in sensitivity, linearity, dynamic range, and bandwidth needed to be made to existing SQUID electronics to achieve the required performance for the FDM system. In this chapter we describe the basic principles of operating of a SQUID, motivate the choice of 4 K SQUID amplifier used in our FDM system and outline in detail the custom 300 K feedback electronics developed for our FDM system.

5.1 Theory of Operation

The operation of a SQUID depends on two phenomena of superconductivity: the Josephson effect and magnetic flux quantization in closed superconducting loops. The Josephson effect was first proposed when Josephson postulated that a very weak link, or Josephson junction, in a superconductor (a constriction, a normal metal barrier, or an insulating barrier) could support a phase dependent supercurrent [53]. Provided that the physical scale of the weak link is small compared with the coherence length of the Cooper pairs, the ensemble
wave functions on either side of the junction overlap or couple. This overlap lowers the system energy by the coupling energy $E_J$. The equations that govern the current and voltage of the junction are the Josephson current-phase relation and the Josephson voltage-phase relation:

$$I = I_c \sin \phi \quad (5.1)$$
$$\frac{d\phi}{dt} = \frac{2eV}{h} \quad (5.2)$$

where $I$ is the Cooper pair current flowing across the junction, $I_c$ is the critical current of the junction (this depends on junction geometry, material and temperature), and $V$ is the voltage drop across the junction. Note that the strength of the coupling across the junction is directly related to the critical current: $E_J = \frac{hI_c}{2e}$.

Note that as in all phases in quantum mechanics, the important physical quantity is always a phase “difference”. In particular, the phase of a wave-function at a certain location and time is gauge-dependent. However, the phase used in equation 5.1 must be a gauge-invariant quantity. In general, then, the gauge-invariant phase difference across a Josephson junction $\phi$ is defined to be:

$$\phi = \theta_2 - \theta_1 + \frac{2e}{h} \int_1^2 \vec{A}(\vec{x}, t) \cdot d\vec{l} \quad (5.3)$$

where $\theta_2$ and $\theta_1$ are the wave-function phases on each side of the junction, defined in the same gauge as the vector potential $\vec{A}(\vec{x}, t)$.

### 5.1.1 Josephson Junction Model

The DC behaviour of an ideal Josephson junction is described by the above equations. It has a current carried solely by Cooper pairs, not by quasiparticles (electrons), and no additional parasitics such as resistance or capacitance. In reality, for $T > 0K$, there is a non-zero population of quasiparticles that contribute to current flowing through the junction. In addition, at least for insulating barrier junctions, the junction area is designed to be
large and the thickness small to increase $I_c$, producing a stray shunt capacitance across the junction. A more realistic model of a Josephson junction must take an effective resistive (from quasiparticle current) and capacitance (from junction geometry) into account. The resistively and capacitively shunted junction (RCSJ) model, as shown by the schematic in figure 5.1 includes these elements.

The current flowing through the RCSJ is the sum of the currents through each of the components of the model:

$$I = I_c \sin \phi + \frac{V}{R} + C \frac{dV}{dt}$$

or equivalently,
\[
\frac{\hbar C}{2e} \ddot{\phi} + \frac{\hbar}{2eR} \dot{\phi} = I - I_c \sin \phi
\] (5.5)

This is a reasonably complicated non-linear second order differential equation. However, following [45], we recognize that it is completely analogous to the equation of motion of a particle moving in one dimension (where the one-dimensional displacement is analogous to the phase \(\phi\) across the junction) with mass equivalent to \(m = \frac{\hbar C}{2e}\), a damping coefficient \(\gamma = \frac{\hbar}{2eR}\) and a non-linear force term \(F = I - I_c \sin \phi\) that could be derived from a potential of the form:

\[U = -I\phi - I_c \cos \phi\] (5.6)

The effective "washboard" potential of equation 5.6 is shown in figure 5.2. It can be parameterized by the ratio of the current to the critical current \(I/I_c\). This parameter controls the tilt of the effective potential. Thus, this ratio divides solutions into two classes (we are assuming that the temperature is very close to zero and thus \(kT \ll E_J\) is satisfied). Solutions with \(I/I_c < 1\) have periodic potential wells that trap \(\phi\) at certain values. These solutions are static in phase which implies no voltage drop across the junction. The RCSJ can support a Cooper-pair supercurrent less than \(I_c\).

For \(I/I_c > 1\) the solutions are slightly more complicated. We can parameterize the effect of the capacitance (mass analog) with the parameter \(\beta_c\):

\[\beta_c = \frac{2eI_cR}{\hbar}RC\] (5.7)

For \(\beta_c \ll 1\) the junction is said to be over-damped (friction term dominates over inertia term). For \(I/I_c > 1\) the phase advances (particle moves down the "washboard" potential) and the non-zero \(\dot{\phi}\) implies a voltage drop:

\[V = R(I^2 - I_c^2)^{1/2}\] (5.8)

Thus, the over-damped solution in the low temperature limit produces the I-V response
shown in figure 5.3. For larger $\beta_c$, however, the I-V response shows hysteresis [46]. In this case once the phase (particle) has begun changing (displacing), the inertial term, the capacitance (mass) prevents the phase (particle) from being trapped in one of the washboard ripples. The RCJS can support a voltage drop even when $I$ is reduced below $I_c$, resulting in a hysteretical I-V response. Typically the RCJS is shunted with a damping resistor that is much smaller than the normal conductance of the RCJS. This reduces $\beta_c$ and avoids hysteresis.

5.1.2 DC SQUID

The DC SQUID is a device that uses a pair of Josephson junctions connected in parallel with superconducting leads. Figure 5.4b shows the simple schematic of a DC SQUID (each junction is meant to actually be a RCSJ). The strength of the DC SQUID design is that a
magnetic flux passing through the loop containing the junction affects the phase difference across each of the junctions. This in turn changes the effective total critical current $I_T$ of the DC SQUID and thus the DC SQUID I-V response. The DC SQUID thus transduces a magnetic flux to an electrical signal. In practice, a many-turn superconducting coil is usually placed above the DC SQUID loop with an insulating layer between it and the SQUID loop. Passing a current through this coil induces a magnetic flux through the SQUID loop and allows the DC SQUID to be integrated into a larger electrical system.

We want to quantify this dependence of critical current on magnetic flux. There are separate (but overlapping) ensemble wave-functions for the Cooper pairs above and below the junction pair. If we assume that the current deep inside the superconductor is close to zero, then we know $\nabla \theta = -2e\vec{A}/\hbar$ where $\theta$ is the phase of the superconducting wave-function. Then we can integrate this phase in two paths: from point 1 to 2 in figure 5.4a and from point 3 to 4 in figure 5.4a [46]
This equation relates the partial line integral of the phase gradient around the complete loop to the partial line integral of vector potential. The difference between the complete line integral and the remaining segments must have an equivalent relationship:

$$\oint \vec{\nabla} \cdot \vec{dl} = -\frac{2e}{\hbar} \oint_{1}^{2} \vec{\nabla} \cdot \vec{dl} - \frac{2e}{\hbar} \oint_{3}^{4} \vec{\nabla} \cdot \vec{dl}$$ (5.9)

The closed loop line integral of $\nabla \theta$ must be a multiple of $2\pi$ and the closed loop line integral of the vector potential is just the magnetic flux through the loop. Using the gauge-invariant form of the phase difference across a Josephson junction from equation 5.3, the

$$\oint \vec{\nabla} \cdot \vec{dl} - (\theta_{1} - \theta_{4}) - (\theta_{3} - \theta_{2}) = -\frac{2e}{\hbar} \oint A \cdot \vec{dl} + \frac{2e}{\hbar} \oint_{1}^{4} A \cdot \vec{dl} + \frac{23}{\hbar} \oint_{2}^{3} A \cdot \vec{dl}$$ (5.10)
relationship between the phase differences across the two junctions and the magnetic flux is then:

\[ \phi_2 = \phi_1 + 2n\pi - 2\pi \frac{\Phi}{\Phi_0} \] (5.11)

where \( \Phi_0 = h/2e \). The current flowing through the DC SQUID, \( I_S \) is the sum of the two currents through the junctions \( I_S = I_1 + I_2 \) which, using equation 5.11 can be expressed as:

\[ I_S = I_{c1} \sin \phi_1 + I_{c2} \sin \left( \phi_1 - 2\pi \frac{\Phi}{\Phi_0} \right) \] (5.12)

where \( I_{c1} \) and \( I_{c2} \) are the critical currents of the individual junctions (which, for generality, we assume to be different). To find the critical current of DC SQUID as a whole, we maximize \( I_S \) as a function of \( \phi_1 \) at a given input flux \( \Phi \):

\[ I_{Sc}(\Phi) = \left( (I_{c1} - I_{c2})^2 + 4I_{c1}I_{c2} \cos^2 \left( \frac{2\pi \Phi}{\Phi_0} \right) \right)^{1/2} \] (5.13)

Note that this equation is essentially identical to the intensity pattern formed when coherent light from two slits separated by a distance \( d \) is combined at a far image. The flux through the loop is analogous to the \( x \)-position of the image and the slit separation \( d \) is analogous to the flux quantum.

Figure 5.5 shows a simple calculation of this familiar interference pattern assuming the individual junction critical currents are identical. The critical current is periodic in the value of the magnetic flux threading the junction: the critical current is a maximum at integer multiples of a magnetic flux quantum and a minimum at half-integer multiples. This flux-dependence is the key to the utility of a DC SQUID and allows it to act as a transducer between a magnetic flux signal and an electrical signal.

In this simple calculation, the critical current is actually identically zero at half-integer multiples of \( \Phi_0 \). Practical SQUID devices do not show a vanishing critical current at half-integer multiples of \( \Phi_0 \) for two reasons. The first is that it is rare that the individual junctions have identical critical currents. The second, and more important, reason is that
we have ignored the self-inductance $L$ of the SQUID loop itself in deriving equation 5.13. The flux $\Phi$ in this equation is the net flux threading the SQUID loop which, for $LI_c \ll \Phi_0$ is essentially the external flux. However, when this condition is violated, a substantial induced flux is developed by circulating currents in the DC SQUID. This has the effect of decreasing the modulation depth of the critical current between integer and half-integer multiples of $\Phi_0$.

Figure 5.6 shows the measured I-V curve for the DC SQUID used in the FDM development, a series array manufactured by the National Institute of Standards and Technology (NIST). The modulation of critical current is clear from the data: the critical current is maximized at integer values of applied flux.

If a DC SQUID is current-biased just above $I_{sc}$, that is at the transition from the superconducting state to the normal state, then the SQUID will develop a flux-dependent
Figure 5.6. Measured IV curve for NIST SQUID Array at integer and half-integer flux quanta.

Voltage at the output. This $V - \Phi$ relation is periodic in flux, a direct consequence of the periodicity of equation 5.13. Figure 5.7 shows a measurement of such a $V - \Phi$ relation for the same series array NIST SQUID.

With an input coil that converts an electric current to a magnetic flux, the DC SQUID becomes a current-to-voltage transducer. It is the heart of the FDM readout system: the current through a set of bolometers is passed through the input coil of a DC SQUID which in turn produces an output voltage that can be directly coupled to room temperature readout electronics (DC SQUIDs are sometimes operated with a voltage bias, producing a flux-modulated current at the output. To simplify the room-temperature electronics, we choose to couple a flux-modulated output voltage instead of a current to these electronics and thus we operate our SQUIDs with a current bias).
5.2 SQUIDs for the FDM

In choosing the appropriate DC SQUID technology for our FDM system, there are several important SQUID parameters to define and consider:

- $M$ - mutual inductance between input coil, SQUID loop
- $\frac{\partial V}{\partial \Phi}_{\text{max}}$ - the maximum slope of the $V - \Phi$ relation
- $R_n$ - normal impedance of SQUID
- $I_{Sc}$ - critical current of SQUID

The mutual inductance between the SQUID input coil and the SQUID loop is given by $M = \alpha \sqrt{L_{in}L_{sq}}$ where $\alpha$ is a geometry-dependent coupling constant. The mutual inductance determines how much flux a given current through the input coil produces:
\[ \phi = M i_{in}. \] It is measured in units of henry, although it is often equivalently measured as the amount of current needed to produce a flux quanta: \( \mu A / \Phi_0 \).

The maximum slope of the \( V - \Phi \) relation is a measure of the size of the output voltage modulation of a current-biased SQUID. A larger \( \frac{\partial V}{\partial \Phi} \big|_{max} \) obviously produces a larger voltage output for a given flux input. The peak-to-peak voltage modulation, \( V_{pp} \), is often used as a proxy for maximum slope since it can be measured quickly and easily. Then, for SQUID output modulation curves that are close to sinusoidal, \( \frac{\partial V}{\partial \Phi} \big|_{max} \approx \pi V_{pp} \).

Since we operate our SQUIDs as transimpedance amplifiers (convert input currents from a set of sensors to output voltages), the mutual inductance and the maximum slope of the \( V - \Phi \) relation are often combined into a transresistance measured in ohms:

\[ Z_{sq} = M \frac{\partial V}{\partial \Phi} \big|_{max} \quad (5.14) \]

The transresistance of a SQUID is flux-dependent. The next section will outline the electronics we developed to instrument our DC SQUIDs. We usually operate the SQUID at the point of maximum transresistance, which tends to be very close to the \( \Phi_0 / 4 \) point in the \( V - \Phi \) relation.

The normal output impedance of a SQUID is typically 1 – 4\( \Omega \). \( R_n \) is almost always determined by the shunting resistors placed across the junction to ensure \( \beta_c < 1 \). The critical current of a SQUID, \( I_{Sc} \), varies widely depending on the specific Josephson junction geometry. It is typically between 10 – 100 \( \mu A \) which easily satisfies \( E_J \ll kT \) for operation at 4.2K.

### 5.2.1 SQUID Requirements

Several considerations strongly influence the choice of SQUID for our FDM system. We connect the output of the SQUID directly to room temperature electronics which have, at a minimum, an input voltage noise of 1 nV/\( \sqrt{\text{Hz}} \). The room temperature leads connecting the SQUID output to the room temperature electronics also develop voltage noise that is a
Table 5.1. Potential SQUID candidates for the FDM system (the Jena-3 SQUID has two coils coupled to the SQUID washer each with a different mutual inductance).

<table>
<thead>
<tr>
<th>SQUID</th>
<th>Jena3 (strong coil)</th>
<th>Jena3 (weak coil)</th>
<th>SEIKO D</th>
<th>HYPRES</th>
<th>NIST 8-turn</th>
</tr>
</thead>
<tbody>
<tr>
<td># in series</td>
<td>1</td>
<td>1</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Lin (nH)</td>
<td>320</td>
<td>20</td>
<td>200</td>
<td>160</td>
<td></td>
</tr>
<tr>
<td>M (nH)</td>
<td>9</td>
<td>0.230</td>
<td>0.09</td>
<td>0.07</td>
<td>0.08</td>
</tr>
<tr>
<td>Ic (µA)</td>
<td>12.4</td>
<td>12.4</td>
<td>26</td>
<td>&gt; 150</td>
<td>85</td>
</tr>
<tr>
<td>Rn (Ω)</td>
<td>3.7</td>
<td>3.7</td>
<td>200</td>
<td>300</td>
<td>80</td>
</tr>
<tr>
<td>spec. Vpp (µV)</td>
<td>37</td>
<td>37</td>
<td>2000</td>
<td>10000</td>
<td>4500</td>
</tr>
<tr>
<td>meas. Vpp (µV)</td>
<td>0</td>
<td>9</td>
<td>800</td>
<td>6800</td>
<td>3500-4500</td>
</tr>
</tbody>
</table>

fraction of 1 nV/√Hz. We require that the TES bolometer noise level, after amplification by the SQUID, easily override this voltage noise level. From chapter 3, we expect that this noise level is 10 pA/√Hz. This sets a firm lower limit of 100 Ω on SQUID transimpedance that we can tolerate in our system. We would like to easily override the room temperature noise. Thus we require SQUID transresistances of 400 – 500 Ω. In the next section we will also see that when we operate our SQUID with room temperature electronics to extend its dynamic range, transresistances of close to 500 Ω are required.

A second important consideration is the expected current level from a single TES bolometer. The bolometer design discussed in chapter 3 requires a bias power of at least 60pW. The normal resistance of a TES bolometer is close to 1Ω. For a bolometer operating at \( R = 0.25R_n \), the current needed to supply this power is 15 µArms. Room temperature feedback electronics, discussed in detail in the next section, is used to extend the dynamic range of the linear portion of the \( V - \Phi \) relation from a fraction of a flux quantum to several flux quantum. Thus we want to choose \( M \) such that 12 µArms corresponds to less than 1 \( \Phi_0 \).

5.2.2 SQUID Candidates

Table 5.1 shows a list of the SQUID candidates tested for use in the FDM system along with their relevant properties. In particular the last two columns of table 5.1 contain the manufacturer specified \( V_{pp} \) and the actual achieved \( V_{pp} \) in Berkeley. Notice that there is a
significant difference in most of the SQUID candidates between the specified and achieved values of $V_{pp}$.

The first SQUID cooled in Berkeley was the single Jena-3 SQUID [54]. This SQUID contains two coils that couple with different strengths to the SQUID washer. We had significant difficulty achieving anything close to the specified $V_{pp}$ values with the Jena-3 SQUID, especially using the strongly coupled coil. When operating with the strong coil, the SQUID was extremely sensitive to radio-frequency interference (RFI). Even when testing was moved into an RFI-shielded room, no discernible flux-modulated voltage could be seen. The tight geometrical coupling between the Jena-3 strong coil and SQUID washer produces a sizeable stray capacitance between them. RFI present on the leads connected to the Jena-3 strong coil is thus almost directly placed across the Josephson junction pair. The hypothesis is that even small amounts of RFI can effectively heat the junction pair and destroy any voltage modulation (the relation $E_J \ll kT$ is violated). Operating with the weakly coupled coil produced a measurable voltage modulation, but still only a fraction of the modulation specified by Jena.

The achieved transresistance with the Jena-3 SQUID is only $\approx 3 \, \Omega$, two orders of magnitude below our requirements. In addition, both the strongly and weakly coupled coils have mutual inductances that produce much more than a flux quantum when a single TES bolometer carrier is present at the input of the SQUID. The mutual inductance needs to be smaller by a factor of several while the overall SQUID transresistance needs to be larger by about two orders of magnitude.

A natural way of approaching these conflicting requirements is through the use of series-array SQUIDs (SSAs) [55]. Series array SQUIDs consist of many single DC SQUIDs wired in series, as shown by the schematic in figure 5.8. Each single DC SQUID element couples through the same mutual inductance $M$ to the input coil of the SSA. The power of SSAs is that voltages from each individual DC SQUID element add in series, greatly increasing the transresistance of the device. In principle a SSA allows us to keep $M$ low enough that a single bolometer carrier is well within one $\Phi_0$ while increasing $V_{pp}$ by a large enough factor to produce a final transresistance of $400 - 500 \, \Omega$. 58
The first SSA cooled in Berkeley was a 100-element array manufactured by SEIKO Corporation [56]. The SEIKO SSA had an almost ideal mutual inductance for the FDM system. However, the $V_{pp}$ of this device was too small to produce a transresistance that met our design criterion.

We evaluated SSAs from two other vendors, HYPRES [57] and NIST [58]. We measured a maximum transresistance in excess of $730 \, \Omega$ for the HYPRES SSA and close to $500 \, \Omega$ for the NIST SSA. Both of these transresistances are close to that specified by the vendor and more than meet our design criterion. However, the HYPRES SSA exhibited severe deviations from smooth sinusoidal behaviour in the $V - \Phi$ relation. This sub-structure consisted of distortions as severe as vertical jumps in the $V - \Phi$ relation and produced input current noise that was higher than our design requirements. The combination of high transresistance and a clean $V - \Phi$ relation motivated our choice of the NIST SSA as the SQUID technology in our FDM system.

Figure 5.8. Schematic of Series Array SQUID.
5.2.3 Series-Array SQUIDs

Noise Advantage

Although series-array SQUIDs are, by design and for the majority of the time in practice, viewed as a single SQUID with \( n \) times the \( V_{pp} \), there are several important differences that should be noted. In an important respect, a SSA gives a significant advantage over a single SQUID: the increased \( V_{pp} \) comes with very little noise penalty. The output noise from the individual SQUIDs in the array add in quadrature and thus a \( n \)-series SQUID array has \( \sqrt{n} \) times the output noise. However, the output voltages from the individual SQUIDs in the array add coherently and thus an \( n \)-series SQUID array has \( n \) times the \( V_{pp} \), increasing the signal-to-noise (defined as the output voltage divided by the output noise) by a factor of \( \sqrt{n} \). This advantage allows SSAs to achieve an input current noise well below 5 pA/√Hz for mutual inductances that meet our design requirement. In fact, the combination of high transresistance, low input current noise, and a mutual inductance that maps a bolometer carrier to less than a flux quantum cannot be achieved with current single SQUID technology. The SSA provides the needed performance for the FDM system.

Magnetic Flux Gradients

However, there are two major disadvantages in operating SSAs over single SQUIDs. The first is increased sensitivity to trapped flux and background magnetic fields. Since the voltages from the \( n \) SQUIDs must add coherently, they must be in identical flux environments. The presence of a slightly non-uniform background field or trapped flux can both produce a gradient in magnetic flux across a SSA. If this gradient is large enough compared to a flux quantum, the flux environment is not identical across the SSA, the effective \( V_{pp} \) is decreased, and the performance of the SSA is severely compromised.

In order to reduce the effect of non-uniform background fields, the SSA must be shielded. For the FDM system development the SSA is cooled and operated in a niobium (Nb) shield at 4K. The Nb shield, aside from two small holes for wiring, completely encloses the SSA.
A Nb superconducting shield has the effect of "freezing" in place the magnetic field that is present as the shield is cooled through its superconducting transition (Nb is a type II superconductor) [45]. Thus, the Nb shield guards against time-varying magnetic fluxes and flux gradients across the SSA. To reduce the absolute value of the magnetic field present at the SSA as the Nb shield cools, the cryostat is enclosed in a cylindrical ferromagnetic shield (\(\mu\)-metal) as the Nb shield cools from 77K to 4K. A cold version of the ferromagnetic shield was subsequently developed for conveniently cooling larger receivers [59].

With the combination of ferromagnetic and superconducting shields, an SSA can be routinely cooled and operated in any laboratory setting. However, a SSA can potentially "trap" magnetic flux through one or more of the \(n\) single SQUIDs in the series. Trapped flux, like a non-uniform background field, destroys the flux coherence and thus the performance of the array. The typical ways in which SSAs trap flux include driving large currents through the SSA input coil or junctions because of electrostatic discharge through a poorly grounded receiver or readout electronics, cooling the SSA with large junction or flux current biases present, or rapidly and non-uniformly heating and cooling the SSA.

In the early development with SSAs, we found we were particularly susceptible to both trapped flux and RF interference. Unfortunately the symptom of both of these is a decreased \(V_{pp}\). Thus, in order to differentiate the cause of the compromised \(V_{pp}\) of an SSA (i.e. either trapped flux or severe RFI), we use a technique called zero-voltage biasing to test for the presence of trapped flux. To zero-voltage bias a SSA, we turn down its junctions’ current bias until the SSA is unresponsive to applied flux over the majority of a flux quantum (i.e. the current bias is set just above \(I_{Sc1}\), the lower critical current of the SSA). If the individual SQUIDs are not all in an identical flux environment, a fraction of them will "turn on" and produce a voltage at the output when the majority of the array is still "off". Small bumps showing this response at the zero baseline of the zero-voltage bias \(V - \phi\) relation is an excellent diagnostic for the presence of trapped flux and is important in separating the effects of trapped flux and RFI in degrading the performance of SSAs. Figure 5.9 shows an examples the zero-voltage bias \(V - \Phi\) relations for a SSA with no trapped flux and the same SSA with a moderate amount of trapped flux.
To guard against the effects of trapped flux, we place a 100 Ω heater near the cold SSA. When the SSA traps flux, a current is dissipated in the resistor which heats the SSA above its superconducting transition. If the SSA is heated above $T_c$ and then subsequently is let cool with all of the attached room-temperature electronics quiescent with no currents flowing through the SSA, the trapped flux is easily removed.

**$V - \Phi$ substructure**

The second major disadvantage of SSAs is the presence of highly non-linear regions in the $V - \phi$ relation, which we call substructure in the $V - \phi$ relation [60]. The substructure tends to get more severe with the number of turns in the input coils coupled to the individual SQUID washers and the number of individual SQUIDS in the SSA. High frequency oscillations in the output of the SSA at frequencies from tens to hundreds of MHz (but
usually below the Josephson frequency) are typically observed when an SSA is operated in the region of severe non-linearity.

The most likely mechanism for inducing this substructure has two components: the input inductance of the input coil of the SSA and the parasitic capacitance between this coil and the SQUID washer form a very high Q LC resonator that is excited by the input current noise of the SSA and produces an output voltage oscillation. Longer input coils and tighter the capacitive couplings between the input coil and the SQUID washers produce lower frequency, and thus more pronounced, output oscillations. The second component of the mechanism involves the coupling of this output oscillation, through the parasitic capacitance, to the input coil of the SQUID, producing a feedback loop in the SSA itself [61]. Simulations and experimental evidence support this model. It is important to note that structure is not inherent to SSAs. Extremely tight-coupling between input coil and SQUID washer produces structure even in single SQUIDs. However, because of the large number of SQUIDs in series in the SSAs used in the FDM system, structure becomes important for much less tightly coupled input coils. Figure 5.10 shows the measured $V - \Phi$ relation for a 100-element SSA fabricated by HYPRES. Notice that the SSA has a relatively large $V_{pp}$ but also has severe structure.

Two common strategies are employed to alleviate the problem of structure in SSAs. The first involves inserting damping resistors between individual turns in the SSA input coil. This effectively lowers the Q of the LC resonance formed at the input coil and damps the large oscillating currents that produce structure. Although using damping resistors has the disadvantages of lowering the SSA bandwidth and decreasing its sensitivity, the success of this technique in producing $V - \Phi$ relations free of structure outweighs these disadvantages. A second strategy used to alleviate structure in SSAs is to either increase the center-to-center spacing of the individual SQUIDs on the SSA chip or insert an inductive meander in between the outputs of individual SQUIDs. This decreases the coupling of these oscillating outputs between SQUIDs and ensures that the SQUIDs act individually. The SSAs used in the FDM system use both of these strategies and have clean structure-free $V - \Phi$ relations as shown in figure 5.7.
5.3 SQUID Electronics

Notice that the $V - \Phi$ relation of the NIST SSA shown in figure 5.7 is highly nonlinear for flux inputs that are even a small fraction of $1\Phi_0$. In addition, the SSA has a limited dynamic range over which its response is monotonically changing with applied flux. In order to use the SSA effectively as a cold transimpedance amplifier, its linearity must be improved and its dynamic range extended. The usual way of doing this is by operating the SSA in a feedback loop. In particular, room temperature SQUID electronics are used to operate the SSA in a flux-locked loop (FLL). By using electronics to “zero” any applied flux at the input of the SSA, the goals of increasing linearity and dynamic range are achieved.

In addition, the room-temperature SQUID electronics must supply low noise current biases to both the junctions and the input coil to bias the SSA at its operating point.
Finally, a large current is needed to heat the 100 $\Omega$ resistor used to thermally cycle the SSA in the event of trapped flux. In this section we outline the SQUID electronics developed to operate the 100-element SSAs in order to produce a transresistance amplifier that meets the requirements of the FDM system. Detailed derivations of several key relations for our SQUID electronics have been placed in appendix A for reference.

![Traditional FLL circuit designs](image)

5.3.1 Electronics Overview

The usual goal of room temperature SQUID electronics is to operate a SQUID device in a FLL: the flux through the SQUID device is held constant by a combination of the SQUID...
device and its coupled room temperature amplifiers. Two traditional FLL architectures exist: alternating current (AC) and direct current (DC) designs. Figure 5.11 shows basic schematics of the two designs.

In the AC architecture, an AC flux bias, usually in the range of 100 to 500 kHz and with an amplitude of $\Phi_0/4$ is applied to the SQUID coil [62]. In addition, a DC flux bias is applied to operate the SQUID at a point in its $V - \Phi$ relation corresponding to $n\Phi_0$ (i.e. an extremum in its $V - \Phi$ relation). The AC flux bias is thus rectified in the output voltage of the SQUID. A lock-in detector referenced to the unrectified AC flux bias will produce zero voltage out. However, if an additional applied flux moves the SQUID away from $n\Phi_0$, the output of the SQUID will have an unrectified component at the AC flux bias frequency. The lock-in will produce an output voltage which can be integrated and fed-back to the SQUID coil as a current to "zero" this additional applied flux. The flux signal is then measured as a voltage at the output of the integrator.

In the simpler DC architecture, a DC flux bias is applied to the SQUID coil to operate the SQUID at the $(n-1/4)\Phi_0$ point in its $V - \Phi$ relation. A room temperature amplifier directly measures the output voltage of the SQUID and its offset is adjusted to produce zero voltage out at the $(n-1/4)\Phi_0$. An additional applied flux moves the SQUID away from $(n-1/4)\Phi_0$ which produces a voltage at the output of the room temperature amplifier. This voltage is integrated and fed-back to the SQUID coil as a current to "zero" this additional applied flux. Like the AC architecture, the applied flux signal is measured as a voltage at the output of the integrator.

In both architectures, the strength of the FLL is determined by the closed loop gain. We can imagine breaking the feedback loop at some convenient point and applying a small input current to the SQUID, $I_s$. The closed loop gain is then defined as the ratio between the resulting feedback current, $I_f$, and the input current $I_s$:

$$A_{cl} = \frac{I_f}{I_s} = \frac{Z_{\text{squid}} A_v}{R_f} \quad (5.15)$$

The loop gain depends on the transresistance of the SQUID, $Z_{\text{squid}}$, the total voltage
gain of the room temperature electronics, $A_v$, and the feedback resistance, $R_f$. As we will derive, $A_{cd}$ determines the extension in dynamic range, increase in linearity, and decrease in input impedance of our SQUID and room temperature electronics.

The AC architecture has the advantage of immunity to slow gain variations in the SQUID and room temperature electronics. Any gain variation on time scales slower than the fast 100 to 500 kHz AC modulation does not effect the FLL. The price of this immunity, however, is increased circuit complexity and a bandwidth limited by the modulation time scale. Since we are biasing our TES bolometers with a sinusoidal voltage, the FDM system is inherently immune to gain variations on time scales slower than the sensor bias frequency and the simpler, higher bandwidth DC architecture is preferred.

Commercially available SQUID electronics with the DC architecture, however, do not have sufficient performance for the FDM system. For reasons discussed in chapter 6, we bias our TES bolometers at frequencies of 300 kHz to 1 MHz. Commercial SQUID electronics that approach the required bandwidth do not have sufficient dynamic range to measure a set of multiplexed bolometer carriers. The main performance limit is the integration stage of the system.

Any high-bandwidth high-gain negative feedback circuit is susceptible to positive feedback and oscillation if phase shifts are not well-controlled and designed around. Parasitic capacitances in the amplifiers and wiring in any feedback chain commonly contribute significant phase shifts at high frequencies. For example, a 50 pF parasitic wiring capacitance in a 100 Ω twisted pair produces 45° of phase shift at 30 MHz. If the phase shift becomes 180° (shifting negative feedback to positive feedback) at a frequency at which the feedback circuit has closed loop gain greater than unity, the feedback circuit will oscillate at this frequency, severely reducing the dynamic range and sensitivity of the system. The design rule for feedback circuitry is to maintain at least 45° of phase margin (that is, 45° away from positive feedback) at all frequencies where the closed loop gain of the feedback circuit is above unity. In other words, the feedback signal should acquire at most, a 135° phase lead or lag before the gain falls below unity [63].
The integrator exists in the SQUID electronics to provide a reduction in the high-frequency feedback circuit gain while simultaneously maintaining a well controlled phase shift until the gain reaches unity. Thus, the integrator ensures that the total voltage gain of the warm electronics, which has a DC value of $A_v$, follows the equation:

$$A_v(\omega) = \frac{A_v}{1 + i\omega\tau}$$  \hspace{1cm} (5.16)

where $\tau$ determines the integrator bandwidth: $f_{\text{int}} = 1/2\pi\tau$.

In practice, this means that the integrator roll-off frequency, or bandwidth (defined to be the frequency at which the gain has been reduced by 3dB) must be well below any other bandwidth limiting components.

For commercially available SQUID electronics with multiple amplification stages the 3dB frequency of the integrator, by design, puts a severe limit on the bandwidth of the FLL. For the FDM system, we developed custom SQUID electronics without an explicit integration stage in order to increase the bandwidth of our FLL.

### 5.3.2 FDM SQUID Electronics Design

For an eight-channel FDM system, our SQUID electronics have to satisfy the following requirements:

- input current noise < 10 pA/$\sqrt{\text{Hz}}$
- bandwidth $\approx$ 1 MHz
- dynamic range of 8 times 12 $\mu$A$_{\text{rms}}$
- input impedance $\ll 1 \Omega$

The first requirement ensures TES bolometer limited sensitivity, the second allows us to bias our TES bolometers up to 1 MHz, and the third allows us to simultaneously measure eight typical bolometer carriers without exceeding the dynamic range of the FLL. Finally,
the fourth requirement ensures that the SQUID readout does not compromise the voltage bias of the TES bolometers and that it is acting as a nearly ideal ammeter.

Figure 5.12. Simple schematic showing the custom FLL developed for the FDM system.

**Single Amplifier**

Figure 5.12 contains a simple schematic showing our custom FLL design. It is very similar to figure 5.11b. One of the main differences is that the room temperature signal processing consists of a single high gain-bandwidth (GBW) operational amplifier (op-amp) whose output voltage is fed directly back to the SQUID input coil. The initial development of the FLL was done with an Analog Devices op-amp which has a GBW of 100 MHz (AD797). The current production version of the FLL contains a Burr-Brown op-amp with a GBW of 3.4 GHz (OPA847). The gain roll-off and phase control is accomplished with the internal compensation built into the op-amp package. This provides a simple single-pole RC roll-off above a 1 MHz bandwidth.
Shunt Feedback

The second difference shown in figure 5.12 is the way that the feedback current is applied to the SQUID. The traditional SQUID electronics architectures, both AC and DC, apply the feedback current to a separate SQUID coil from the one used to measure the input current. Our design, however, applies the feedback current to the same coil used to measure the input current. This “shunt feedback” lowers the input impedance of the SQUID by “shunting” the input current from the input coil. This is a key design feature: since the input inductance of the NIST SSA is 150 nH (see Table 5.1), at 1 MHz the inductive reactance is close to 1 Ω. Without the reduction of input impedance gained from shunt feedback, the SQUID input inductance would completely destroy the voltage bias of the TES bolometer. Appendix A explains the effect of shunt feedback on input impedance in more detail.

Figure 5.13. Calculation of the expected phase shift of a feedback signal. The phase shift is shown for frequencies up to 50 MHz assuming a round trip time delay of 4ns.
Minimize Wiring Delays

Although the use of a single high GBW op-amp allows us to control the phase shifts that arise from multiple amplifier and integrator stages, there is another limitation to the bandwidth of our FLL. The 300K room temperature amplifier and the 4K SSA must necessarily be separated by wiring of a finite length since they must be maintained at different temperatures. Time delays are incurred when the signal propagates from the 4K SSA output along these wires to the 300K room temperature amplifier and back to the input coil of the SSA along these wires. Although these time delays are minimal at the bolometer bias frequencies which are below 1 MHz, they can produce sizable phase shifts at frequencies where the FLL still has gain greater than unity. For instance, figure 5.13 shows the phase shift of the feedback signal for frequencies up to 50 MHz, the frequency at which the FLL gain is nominally unity assuming a round trip signal propagation delay of 5ns (around 25-30cm wire length connecting the 4K component to the 300K component). The $90^\circ$ phase shift from the amplifier roll-off has not been included in figure 5.13. At 50 MHz the phase shift from wiring alone with this propagation delay is $72^\circ$, resulting in a phase margin of only $20^\circ$.

The phase shift acquired from the wiring between 4K and 300K puts a severe constraint on the GBW product of the FLL. For a closed loop gain of 50 and a bandwidth of 1 MHz, the nominal specifications of the FDM FLL, this constrains the wire length connecting 4K and 300K to be less than 0.18 m. Several schemes to extend the GBW product of the FLL are currently under investigation. These involve moving a gain stage from 300K to 4K to reduce the wire length of the FLL and will be discussed in detail in chapter 8.

Lead-lag Filter

We rely on the single-pole roll-off of the first stage amplifier to control the gain and phase shift of the FLL at frequencies above the highest frequency we use to readout TES bolometers. However, the gain can be decreased with another filter stage without incurring a significant phase penalty.
We use a RC shunt, or lead-lag, filter at the output of the SSA to decrease the high-frequency gain $>3\,MHz$ of the SQUID electronics while preserving the gain in the FDM readout bandwidth $<1\,MHz$. Appendix B describes the lead-lag filter in more detail.

When integrating the SQUID electronics with TES bolometers and their readout wiring, the lead-lag filter proved essential. The parasitic resonance in the tuning inductor chips, discussed in more detail in chapter 6, induces an oscillation of the FLL at $\sim 22\,MHz$ when the lead-lag filter is absent. The additional gain reduction above the FDM bandwidth provided by the lead-lag filter removes this oscillation.

**Readout Noise**

Several terms contribute to the readout noise of the SQUID electronics. The SSA itself has an input noise current of 1-2 $pA/\sqrt{Hz}$. The 300K amplifier has an input noise voltage of 1 $nV/\sqrt{Hz}$ and the wiring connecting 300K to 4K has a resistance that produces a fraction of a $nV/\sqrt{Hz}$ (we assume that the effective wiring temperature is close to 150K to estimate the level of this noise). The feedback resistor at 300K is directly connected to the input coil of the SSA and produces $\sqrt{4kT/R_f}$ of current noise.

The readout noise of the SQUID electronics is usually measured as an effective current noise at the input coil of the SSA. The SSA input current noise and the feedback resistor current noise add in quadrature. The voltage noise from the wiring and the 300K amplifier is referred back to the input coil as a current by dividing by the transresistance of the SSA (typically 400 to 500Ω). Table 5.2 shows the expected contributions to the SQUID noise from the different components. With a SQUID transresistance of 400 $\Omega$ and $R_f = 5000\,\Omega$, we expect the SQUID electronics to have a current noise of 3.9 $pA/\sqrt{Hz}$.

Table 5.2 shows all the estimated contributions to the readout noise are comparable in size. Maintaining suitable readout noise in the SQUID electronics thus puts strong restrictions on all of the different components of the SQUID electronics. For example, if the first stage amplifier input voltage noise grows above 1 $nV/\sqrt{Hz}$, it will begin to dominate the readout noise and the sensitivity of the SQUID electronics will quickly decrease. Decreasing
Table 5.2. Estimated contributions to the SQUID electronics noise. A SSA transresistance of 450 Ω is assumed.

<table>
<thead>
<tr>
<th>source</th>
<th>voltage noise (nV/√Hz)</th>
<th>current noise (pA/√Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300K amplifier</td>
<td>1.08</td>
<td>2.55</td>
</tr>
<tr>
<td>leads</td>
<td>3.5e-10</td>
<td>0.9</td>
</tr>
<tr>
<td>$R_f$</td>
<td>N/A</td>
<td>1.8</td>
</tr>
<tr>
<td>SSA</td>
<td>N/A</td>
<td>2</td>
</tr>
<tr>
<td>total</td>
<td>N/A</td>
<td>3.9</td>
</tr>
</tbody>
</table>

The feedback resistor $R_f$ (in order to increase the loop gain of our system according to equation 3.10) below 5K will also quickly decrease the sensitivity of the SQUID electronics. Finally, if the noise of the SSA itself increases above its current level of around 2pA/√Hz, the sensitivity of the SQUID electronics will suffer.

5.3.3 Performance

![SQUID Electronics Noise](image)

Figure 5.14. Noise of SQUID electronics measured at 300 kHz.
The SQUID electronics developed for the FDM system meet our design requirements. Figure 5.14 shows a measurement of the current noise of the SQUID electronics, measured at a frequency of 300 kHz. The readout noise has a flat power spectrum in a bandwidth of 400 Hz around the 300 kHz measurement frequency at a level of 4.5 pA/$\sqrt{\text{Hz}}$. According to table 5.2, we expect about 3.9 pA/$\sqrt{\text{Hz}}$ of noise which agrees very well with the measurement. In addition, the SQUID electronics meets our requirement of $i_n < 10$ pA/$\sqrt{\text{Hz}}$.

![Dynamic Range of FLL](image)

Figure 5.15. Measured open and closed loop $V - \Phi$ relations. Open loop is shown in blue, $R_{fb} = 10 \, k\Omega$ is shown in red, and $R_{fb} = 5 \, k\Omega$ is shown in black.

The closed loop gain, defined in equation 3.10, determines the dynamic range of the FLL. Appendix A derives a relation for the dynamic range (in terms of input current) and the closed loop gain. With our SSA transresistance of $300 - 500 \, \Omega$, a 300K amplifier gain of 500, and a feedback resistance of $5000 \, \Omega$, we can achieve stable loop gains of 30 to 50.

Figure 5.15 shows the open loop $V - \Phi$ relation (blue) plotted with the closed loop response when locked with $R_f = 10 \, k\Omega$ (red) and when locked with $R_f = 5 \, k\Omega$ (black). Notice that the SQUID response to applied current is linearized and the dynamic range is
Table 5.3. Comparison of estimated and measured dynamic ranges for $R_f = 10 \, k\Omega$ and $R_f = 5 \, k\Omega$. A SSA transresistance of 300 $\Omega$ and a purely sinusoidal $V - \Phi$ relation are assumed for the calculations.

<table>
<thead>
<tr>
<th>$R_f$</th>
<th>measured ($\mu A_{pp}$)</th>
<th>expected ($\mu A_{pp}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10 , k\Omega$</td>
<td>98</td>
<td>131</td>
</tr>
<tr>
<td>$5 , k\Omega$</td>
<td>190</td>
<td>260</td>
</tr>
</tbody>
</table>

greatly extended. The feedback has converted the highly non-linear, low dynamic range SSA into the transresistance amplifier needed for the FDM system. The dynamic ranges measured in figure 5.15 are summarized in table 5.3 along with estimates of the expected dynamic range. The estimates are about 25% higher than the measured dynamic ranges. This slight discrepancy comes from the assumption in the calculation of dynamic range that the $V - \Phi$ relation of the SSA is purely sinusoidal. This is clearly violated by measurements of the $V - \Phi$ relations like the one shown in figure 5.7. However, the dynamic range of 190$\mu A_{pp}$ meets the requirement for an eight-channel FDM system.
Chapter 6

Sinusoidal Bias

The central feature of our FDM system is sinusoidal (AC) biasing of TES bolometers. As outlined in chapter 4, each member of a set of $n$ sensors is biased at a unique frequency, separating the $n$ sensor signals in frequency and allowing them to be summed. AC biasing of TES sensors at frequencies between 300 kHz and 1.2 MHz was pioneered at UC Berkeley and was shown to be completely equivalent to the traditional technique of DC biasing with respect to the performance of TES bolometers.

There are, however, some requirements that this technique demands of the readout system. For example, the stray inductance $L_s$ of the bolometer leads and the wiring connecting the bolometer to the SSA produces a frequency dependent inductive reactance $i\omega L_s$. A stray inductance of 200nH contributes approximately 1 Ω of inductive reactance at 1 MHz. This reactance must be tuned out in order to operate 1 Ω TES bolometers under voltage bias.

This chapter outlines the development of the AC biasing technique for TES bolometers. I motivate the need for cold tuned filters and describe their development and performance. AC and DC biasing of a TES bolometer are compared and I discuss the requirements that must be satisfied to stably operate TES bolometers under voltage bias.
6.1 Tuned Filters

A tuned filter consisting of a cold inductor and capacitor (LC filter) is placed in series with each multiplexed sensor for three reasons. The first is that the LC filter "tunes out" stray reactances that are in series with the TES sensor, allowing the sensor to be cleanly voltage-biased. Second, since the sensors are resistive elements, they contribute a broadband Johnson current noise of $\sqrt{4kT/R}$. The tuned filters bandwidth limit this noise and prevent it from leaking to the other $n$ channels in a multiplexed set. Finally, the tuned filters allow a "comb" of biases to be sent down a single pair of bolometer bias wires which greatly reduces the wire count to the sub-kelvin temperature stages. The tuned filters pass the appropriate bias frequency and suppress the other $n-1$ bias frequencies.

6.1.1 Filter Introduction

An LC filter is a single-pole resonant bandpass filter. The center frequency of its bandpass is determined by the capacitance and inductance that form the filter:

$$f_c = \frac{1}{2\pi \sqrt{LC}}$$  \hspace{1cm} (6.1)

The high frequency cut-off is due to the increasing inductive reactance with increasing frequency and the low frequency cut-off is due to the increasing capacitive reactance with decreasing frequency.

The bandwidth of the LC is determined by the sum of all the dissipative elements in series with the filter. Figure 6.2 shows the expected current flowing across a voltage-biased LCR for three different dissipative terms. The bandwidth is inversely proportional to the size of this dissipative term. For an ideal LC filter (lossless inductance and capacitance) in series with a TES bolometer operating at $R_i$, the bandwidth (defined as the full-width at half-maximum in power) is given by:

$$\Delta f = \frac{R_i}{2\pi L_i}$$  \hspace{1cm} (6.2)
where $L_i$ is the inductance of the LC filter. In practice there also is a small parasitic contribution, $R_p$, to the dissipation from a non-ideal inductance or capacitance which could affect the bandwidth of the LC filter: the total dissipation is then $R_T = R_i + R_p$. The dissipative terms in an LCR circuit are also conveniently summarized by a quantity called the quality factor, or $Q$, of the filter. The $Q$ is defined as the ratio of the center frequency to the bandwidth and is also inversely proportional to the size of the dissipation:

$$Q = \frac{f_c}{\Delta f} = \frac{f_c 2\pi L_i}{R_T}$$  \hspace{1cm} (6.3)

### 6.1.2 Filter Requirements

There are several requirements on the LC filters. The first is that the quality factor ($Q$) of the LC filter must be high enough that the bolometer voltage bias is not compromised. This is equivalent to requiring that the equivalent series resistance (ESR) of the LC filter

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is much smaller than the operating resistance of the bolometer. We require that the LC filter have an ESR that is smaller than the lowest bolometer operating resistance by a factor of at least 6. The Q of the LCR combination is then determined only by the bolometer resistance.

We also must be able to achieve LC filter center frequencies within several percent of design frequencies in order to maintain the desired channel spacing. For stable sensor operation, the bandwidth of the LC filter must exceed the bandwidth of the sensor.

A filter with a realistic inductance and capacitance will not act as a perfect LC filter at all frequencies. Parasitic resonances, due for example to stray capacitance between inductor coils, will alter the transfer function of the LC filter at high frequencies. We require that
parasitic resonance frequencies are well above our highest planned operating frequency of 1 MHz.

Finally, since one LC filter is needed for every multiplexed TES sensor, we require that the physical footprint of the filter does exceed the size of a TES sensor. This size, for a typical 150 GHz pixel, is about 5 cm on a side.

### 6.1.3 Filter Design

**Component Values**

The LC filters are designed for TES bolometers that have an operating impedance of 0.5 Ω and a time constant of $\tau = 0.5$ ms. Operating frequencies range from 300 kHz to 1 MHz. The frequency range is bounded on the lower end by two things: the bias frequency must be much faster than the thermal response time of the TES sensor so that essentially constant bias power is delivered. The LC filter must also have a physical footprint that does not exceed that of the sensor. Inductive and capacitive components begin to grow larger than this limit in order to achieve bias frequencies below 300 kHz. The frequency range is bounded on the upper end by the bandwidth of the SQUID electronics which is designed to be 1 MHz.

The filter bandwidth, given in equation 6.1 must exceed the sensor bandwidth by a factor of 6. Since the sensor bandwidths and operating resistances are all expected to be the same across a large array, the filter bandwidth should be a constant. This means that the same inductance value is used for every LC filter. For 1 ms sensors, the inductance must be < 50 µH.

Since the inductance is fixed, the center frequency is set by the capacitance. To remain in the FDM frequency range for inductances between 10 and 50 µH, capacitances between 0.5 and 20 nH are needed.
Inductors

The LC filters are placed close to the sensors and typically operate at temperatures achieved by a $^3\text{He}$ fridge (250-350mK). The natural choice for a high Q inductor is to take advantage of the low operating temperature and use a superconducting coil. We use a standard inductor design consisting of a superconducting coil deposited on and insulated from a slit square washer (see figure 6.3). This is a proven design for planar SQUIDs [64] and has the advantage of producing minimal stray magnetic field in the plane outside the square washer. This leads to very low mutual inductances between coils placed close together and tight control of inductances. For an $n$ turn coil placed over a square washer with outer diameter $D$ and inner diameter $d$ (provided that the inner diameter is much smaller than the outer diameter) the inductance is very close to:

$$L = n^2 \cdot 1.25\mu_0 d$$

(6.4)

Figure 6.3. Schematic (not to scale) of a simple washer inductor
The coil and washer are thin films of niobium which has no resistive loss at 250mK, allowing the inductors to achieve the required Q for operating TES bolometers.

Spiral inductors can also be placed physically adjacent to one another with very little cross-coupling. The mutual inductance between two inductors is given by $M = k\sqrt{L_1L_2}$ where $k$, the coupling constant, is a geometric factor that measures how well two inductors are coupled. The coupling constant for two co-planar spiral inductors depends only on the radius of the inductors and the center-to-center spacing of the inductors. [65] calculates that for two co-planar spiral inductors in free space (i.e. without square washers) with diameters of 2 mm, the coupling constant is 0.01 for a center-to-center spacing of 1.7 diameters. Square washer inductors should perform even better, and we expect to achieve $k < 0.01$ for center-to-center spacings of 1 to 1.25 diameters.

One final note about the square washer design: the thin insulating layer between the coil and the washer results in a non-negligible parasitic capacitance that effectively shunts the inductor. This produces a parasitic parallel resonance at high frequencies. In the next section I will discuss measurements of this parallel resonance and show that for our design, this resonance is well above the bandwidth of the FDM system.

**Capacitors**

Developing capacitors with the required Q values is much more of a challenge. To achieve appropriate capacitances in a geometry that does not exceed the pixel area, we need either a high-dielectric constant ($\epsilon > 8$) or an extremely thin dielectric layer. Both strategies present challenges.

The candidate high $\epsilon$ dielectric materials tend to have large enough loss that the resulting capacitors have Q values below our requirement. In particular, one of the materials we investigated, Nb$_2$O$_5$, produces Q values a factor of 3 lower than our requirement.

Extremely thin dielectric layers are prone to defects or irregularities. These defects, often called “pin-holes”, usually result in sizable conduction across the dielectric layer which also produces Q values lower than our requirement.
We had some success in developing a hybrid dielectric layer consisting of a very thin (< 400 Å) low $\epsilon$ dielectric with a slightly thicker high $\epsilon$ dielectric then deposited to try to "plug" the pin-holes. However, we discovered that a commercial ceramic technology exists with a very low temperature coefficient and the required Q values. The capacitors are available in surface mount packaging with a chip size about 3 mm X 1.5 mm (for example, Digikey P/N PCC152BNCT-ND).

6.1.4 Filter Performance

We performed two fabrication runs in developing the LC filters. All of the fabrication was performed at Northrop-Grumman. The first fabrication run, which we will call BK1, produced LC chips that each contained eight 40 $\mu$H inductors and eight capacitors. The dielectric for the capacitors is Nb$_2$O$_5$, produced by anodizing a layer of Nb.

The initial bolometer testing, described later in this chapter, showed that the bolometers were faster than expected by a factor of 4 which caused us to increase the LC filter bandwidth by decreasing the inductances of the inductors. A second fabrication run was performed, which we will call BK2. Three types of chip were produced: an L only chip, consisting of eight 15.8 $\mu$H inductors and designed to be used in conjunction with commercial surface mount capacitors, an LC chip consisting of eight 15.8 $\mu$H inductors and eight capacitors with SiO$_2$ as a dielectric, and an LC chip consisting of eight 15.8 $\mu$H inductors and eight capacitors with a hybrid dielectric, SiO$_2$ and Nb$_2$O$_5$.

BK1 Performance

Figure 6.4 shows a picture of the BK1 LC filter chip. The inductors were measured by performing an L/R measurement which verified a 40 $\mu$H inductance within several percent. We measured the Q of each filter on a chip by placing a 0.47 $\Omega$ resistor in series with each of the eight LC filters. As discussed above, each filter has $Q = \omega_0 L/R_T$. We expected the measured Q values to be consistent with a series resistance of 0.47 $\Omega$; instead the Q values were significantly lower, indicating extra series resistance.
We did a substitution test by swapping one of the on-chip capacitors with a commercial surface mount capacitor to confirm that the source of the excess loading was loss in the capacitor dielectric. The dielectric, Nb$_2$O$_5$, has a measured loss tangent $\delta = 0.003$ where $\delta = R_{\text{excess}}/\omega_0 L$ and $R_{\text{excess}}$ is the equivalent series resistance, or ESR of the capacitor. This loss tangent is a factor of at least three higher than what we require for TES bolometer readout. The commercial surface mount capacitors have a loss tangent $\delta < 0.001$.

The inductors of the LC filters are closely spaced on the chip (adjacent inductors are separated by 1.35 diameters). Inductive coupling between physically adjacent inductors needs
to be small as this is a source of crosstalk in the FDM system. Estimating the magnitude of the magnetic coupling between adjacent inductors, using the geometry of the BK1 LC chip, indicates a coupling of $k \sim 0.01$ (1% coupling) which meets our design requirements.

To measure the magnitude of the coupling between physically adjacent inductors, we used the circuit shown in figure 6.5. A pair of inductors is wired in a 4-point configuration. As a current is passed through the primary circuit, we measure the induced current in the secondary circuit which will depend directly on the coupling between inductors.

To infer the coupling constant from this measurement, we used Orcad PSpice (Cadence PSD 13.0 software package) to simulate the behaviour of the circuit in figure 6.5 for a range of $k$ values. Figure 6.6 shows the simulated response (red) and the measured response (black). The measurements match the simulation for a coupling constant of $k \sim 0.01$, in very good agreement with the expected coupling that we calculated.
Figure 6.6. Simulation and measurement of inductive coupling between physically adjacent inductors on BK1 chip.

The final measurement we performed on the BK1 LC chips was a measurement of the transfer function of the LC filter at very high frequencies (> 1.0 MHz). The capacitance between the coil and the washer of the inductors produces a parallel resonance at some frequency that is, by design, pushed to above 1.0 MHz (see figure 6.7).

We measured the transfer function of the filter chip with two channels connected: 568 kHz and 715 kHz. Figure 6.8 shows the amplitude of the transfer function up to 100 MHz. There is good evidence for a parallel resonance at 12 MHz, much above the highest readout resonance frequency (the resonance predicts an effective parallel capacitance of \( \sim 350 \text{ pF} \)).
BK2 Performance - L Chip

The inductor only chips are designed to be used with commercial surface mount capacitor technology to form an LC filter). The inductance is designed to be 15.8 $\mu$H and measurements of the inductors in an LR circuit confirmed the inductance as 15.8 ± 0.5$\mu$H. Figure 6.9 shows a photograph of the inductor only chip.

I measured the Q and center frequency of each inductor plus capacitor by initially placing a 0.47 Ω resistor in series with each of the LC pairs. Each filter has $Q = \omega_0 L/R_T$. In all the measurements, the measured Q agreed very well with the expected Q for a 0.47 Ω load, with excess resistance never exceeding 40 mΩ.

We measured the inductive coupling between physically adjacent inductors, as with the BK1 inductors. Figure 6.10 shows a comparison of the simulated response (red) for a range of coupling constants and the measured response (black). We also measured the coupling between next nearest neighbours (blue). We infer from the measurements a coupling value of $k \sim 0.007$. This is smaller than the coupling in BK1 by 30% which is expected: the BK2
Figure 6.8. Parasitic capacitance of inductor.

design has smaller and more widely spaced inductors. The measurements indicate an upper limit of $k \sim 0.003$ for next nearest neighbour coupling.

Finally, the high frequency transfer function showed strong evidence for a parallel resonance at 19 MHz indicating an effective stray capacitance of close to 180 pF.

**BK2 Performance LC Chip**

As outlined above, we fabricated two sets of LC chips: the first has a capacitor dielectric of only SiO$_2$. The second has a dielectric of SiO$_2$ with a thin Nb$_2$O$_5$ anodization layer. I measured the Q of each filter in both sets of LC chips by again placing a 0.47Ω resistor in series with each of the eight LC filters. For both sets of chips I measured Q values much lower than what would be consistent with the series resistance of 0.47 Ω.
Figure 6.9. Photograph of the BK2 L chip. The chip is 1.9 cm X 0.4 cm.

Figure 6.12 shows the measured (lower trace) and simulated transfer functions for the LC chip with only SiO$_2$. Only four of the expected eight resonances were present (the lowest measured resonance was $\sim 40$ kHz and is not shown). Further, the measured resonances all have full-width half-maxima of $\sim 42$ kHz, indicating a loading in these channels of 5 Ω, at least a factor of 10 higher than the expected loading.

Figure 6.13 shows the measured (lower trace) and simulated transfer functions for the LC chip with both SiO$_2$ and niobium anodization. All eight channels are present, as expected, but the channels all have full-width half-maxima of 30 kHz. This is at least a factor of 8 higher than the expected loading.

Both sets of the LC chips show unacceptable losses. These losses most likely occur in the capacitors’ dielectric, although additional analysis is needed to quantify the losses and allow us to debug this strategy for making lithographed capacitors.
6.2 Performance of AC Bias

AC voltage bias of a TES bolometer must have three properties:

- The bias frequency must be faster than the thermal response time of TES bolometer by at least a factor of 50 (ensures constant bias power delivered to sensor)

- The LC filter bandwidth must exceed the bolometer bandwidth by a factor of 6 (ensures voltage bias, stable electrothermal feedback). Since the LC filter bandwidth is $\Delta f_{LC} = \frac{R_i}{2\pi L_i}$ and the bandwidth of the sensor is $f_c = \frac{1}{2\pi \tau}$, for a given $\tau$, $L_i < R_i \chi \tau$ where $\chi$ is close to 6.

- The stray impedances in series with the TES sensor must be much smaller (factor of at least 5) than the TES bolometer operating resistance (ensures voltage bias)
Violation of any of these properties leads to the cessation of negative electrothermal feedback and thus unstable operation of the TES bolometer. Thermal fluctuations in such bolometers will tend to rapidly grow over only a few thermal time constants of the sensor and the thermistor quickly latches into the superconducting state. When this happens, the responsivity of the TES bolometer to incident radiation goes to zero.

The boundary between stable and unstable operation is not perfectly defined. Our operating definition of stable operation requires that the bolometer not latch superconducting over a typical operation time of several hours and that the bolometer noise closely match that predicted by thermal fluctuations in the bolometer heat link to its bath (see chapter 3).

The above three properties are conservative requirements for stable AC voltage bias. For example, in the second item in the AC voltage bias properties, a range of factors $\chi$, from about 3-6, result in stable operation. We choose $\chi > 6$ to give us margin in the event of a spread in $\tau$ or lower than expected operating resistances, $R_i$. 
Operating resistance rapidly decreases as the TES bolometer enters its transition. The requirements on the AC voltage bias are harder to meet very low in the transition. We typically have 1.0 Ω bolometers and we design the FDM system for stable operation of TES bolometers half-way into their transitions, at 0.5 Ω.

6.2.1 AC vs. DC

Our first test of AC bias of a TES bolometer were conducted on a single sensor without a series LC filter. The LC filter was avoided so we could directly compare AC and DC bias. The test TES bolometer has a normal resistance of 1.0 Ω and a time constant in its transition of 270 µs.

Our test showed successful AC biasing. We biased the sensor at frequencies between 100 kHz and 1 MHz and observed I-V characteristics that matched those obtained under DC bias of the sensor. Figure 6.14 shows a comparison between an I-V curve taken under a 100 kHz bias and one taken under DC bias.
Figure 6.13. Measured and simulated transfer function of LC chip with hybrid SiO$_2$ and Nb$_2$O$_5$ dielectric.

The main difference between AC and DC operation in this test is the position in the transition we can achieve before the bolometer becomes unstable. Using AC bias, we cannot bias the detector as low in the transition as we can using DC bias. The difference is that the stray inductance of the wiring between the bolometer and the SQUID input (between 50 and 100 nH) contributes an impedance in series with the bolometer that degrades its voltage bias. As the bolometer operating resistance decreases, this degradation gets worse until the electrothermal feedback is destroyed and the bolometer latches into its superconducting state. We could operate lower in the transition with a bias frequency of 100 kHz than with a frequency 1 MHz which supports the model of an stray inductive impedance that increases with frequency. This problem is alleviated when we insert an LC filter, which tunes out the stray inductance.

The bolometer was expected to have a time constant of 1 ms, but a measurement of its time constant showed that it was a factor of 5 faster ($\tau = 270 \mu s$). We observe distortions in the AC bias carrier at 100 kHz, indicating that the bolometer is “following” the bias.
Figure 6.14. Comparison of AC and DC bias of TES bolometer.

This distortions disappear at bias frequencies of 300 kHz and above which validates our designed frequency range for the FDM system of 300 kHz to 1 MHz.

6.2.2 Stability of Bias

To test the second requirement for stable operation (the LC filter bandwidth) and gain confidence that $\chi = 6$ gives us margin for stable operation, we measured IV curves for two types of sensor. Sensor A (see table 6.1) was placed in series with two different tuned filters, one with an inductance of 40 $\mu$H and one with an inductance of 20 $\mu$H. Sensor B was placed in series with a tuned filter with 20 $\mu$H inductance and no tuned filter. We estimate the stray inductance from the wiring to Sensor B to be 150 nH.

Table ?? lists the measured sensor parameters and table 6.2.2 lists the results of the stability measurements for these two detectors.

The predicted lowest operating impedances shown in table 6.2.2 match the measure-
Table 6.1. Sensor Parameters.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Time Constant</th>
<th>Normal Resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor A</td>
<td>$270 \pm 30 , \mu s$</td>
<td>$1.0 , \Omega$</td>
</tr>
<tr>
<td>Sensor B</td>
<td>$1.8 \pm 0.4 , ms$</td>
<td>$1.55 , \Omega$</td>
</tr>
</tbody>
</table>

Table 6.2. Stability Measurements

<table>
<thead>
<tr>
<th>Sensor</th>
<th>$L$ of tuned filter</th>
<th>Lowest Achieved Operating $R$</th>
<th>Predicted Lowest Operating $R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor A</td>
<td>$40 , \mu H$</td>
<td>$0.95 \pm 0.04 , \Omega$</td>
<td>$0.86 , \Omega$</td>
</tr>
<tr>
<td>Sensor A</td>
<td>$20 , \mu H$</td>
<td>$0.41 \pm 0.03 , \Omega$</td>
<td>$0.43 , \Omega$</td>
</tr>
<tr>
<td>Sensor B</td>
<td>$20 , \mu H$</td>
<td>$0.18 \pm 0.04 , \Omega$</td>
<td>$0.07 , \Omega$</td>
</tr>
<tr>
<td>Sensor B</td>
<td>$150 , nH$ (no $C$)</td>
<td>$0.50 \pm 0.03 , \Omega$</td>
<td>$0.47 , \Omega$</td>
</tr>
</tbody>
</table>

ments reasonably well for sensor A, which also has a much more precise time constant measurement than sensor B does. I predict a lower operating impedance for sensor B than I measure. Note that the time constant measurement is less precise for this sensor. There is also a possibility that an unknown stray impedance in the bias circuit raised the lowest achievable operating resistance for in this measurement.

However, despite this discrepancy for sensor B, the behaviour of the sensors with different time constants under AC bias agrees with our stability criterion. In general, as expected, fast sensors need a higher tuned filter bandwidth and thus a lower inductance for a given normal sensor resistance.

### 6.3 Carrier Nulling

The AC voltage bias in the FDM system exists only to deliver electrical bias power to the TES bolometer and separate the multiplexed signals in frequency for simultaneous readout. There is no information from the bolometer at the AC voltage bias frequency. As the sensor changes resistance, it amplitude modulates its carrier and the bolometer signal resides in sidebands around carrier frequencies.

An unmodulated voltage bias carrier can be expressed as a current flowing through a resistor:
\[ I(t) = A \cos(\omega_c t + \phi) \] (6.5)

where \( \omega_c \) is the carrier frequency and the carrier amplitude is \( V_b / R \). When the carrier passes through the bolometer, it acquires an amplitude modulation. If the bolometer is measuring an optical signal at frequency \( \omega_m \), then:

\[ I_m(t) = A \cos(\omega_c t + \phi)(1 + a_m \cos \omega_m t) \] (6.6)

The constant unity term in equation 6.6 corresponds to the amplitude of the carrier which is modulated by the modulation term \( a_m \cos \omega_m t \) which corresponds to the bolometer signal. In general, a TES bolometer sees a range of frequencies:

\[ I_m(t) = A \cos(\omega_c t + \phi)(1 + \int_{BW} a_n e^{i\omega_n t} dt) \] (6.7)

where \( a_n \) is the fourier coefficient of the sensor signal at frequency \( \omega_n \) and \( BW \) refers to the bandwidth of the sensor. Usually the power in the bias carrier is much larger than the bolometer signal itself: \( \int_{BW} a_n e^{i\omega_n t} \ll 1 \). By injecting an unmodulated carrier of equal amplitude and 180° out of phase at the input of the SQUID electronics (i.e. immediately after modulation by the detector) we can suppress the carrier signal. This technique, called carrier nulling, greatly reduces the dynamic range and linearity requirements of the FDM readout electronics. We routinely null carriers at the input of the SQUID electronics to a factor of 10 smaller than their original amplitudes.

The nulled signal has a form identical to equation 6.7:

\[ I_m(t) = A \cos(\omega_c t + \phi)(\Gamma + \int_{BW} a_n e^{i\omega_n t} dt) \] (6.8)

where \( \Gamma < 1 \) corresponds to the fraction of carrier that is unnulled. Notice that carrier nulling has not affected the bolometer signal.
To recover the signal from the bolometer, we demodulate by multiplying 6.8 by an unmodulated reference. Defining $\lambda = A(\Gamma + \int_{BW} a_n e^{i\omega_n t} dt)$,

\[ I_m(t) \cos \omega_c t = \lambda \cos(\omega_c t + \phi) \cos(\omega_c t) = \lambda \cos^2(\omega_c t) \cos \phi - \lambda \cos(\omega_c t) \sin(\omega_c t) \cos \phi \quad (6.9) \]

Using the identity $\cos^2(x) = \frac{1}{2}(1+\cos(2x))$ and filtering out frequencies in the multiplied signal above the highest bolometer signal frequency $\omega_n$ but well below the carrier frequency, we recover the bolometer signal:

\[ I_m(t) \cos(\omega_s t) |_{\text{filtered}} = \frac{1}{2} \lambda = \frac{1}{2} A(\Gamma + \int_{BW} a_n e^{i\omega_n t} dt) \quad (6.10) \]
Chapter 7

Eight Channel Test

After demonstrating sinusoidal bias of a single detector, we conducted an eight channel test of our FDM system. In this chapter we will discuss the design and performance of the eight channel test. We will conclude with future directions for the FDM system development, including estimating the limits on channel count with the current architecture and extensions to the current architecture. Chapter 8 discusses one of the most promising extensions to the performance of the SQUID electronics.

7.1 Eight Channel Test: Design

We conducted two versions of the eight channel test. Both tests were successful demonstrations of the FDM technology. They differ only in the specific hardware included in the test which is outlined in table 7.1. Test A was the initial FDM system test and Test B included almost all of the final components planned for the FDM system operation in an optical receiver.

The eight channel test is designed to have two optical TES bolometers, five dark TES bolometers, and a faux bolometer resistor used for calibration. The two optical channels are illuminated with photons from a cold light-emitting diode (LED, Hewlet Packard P/N HP1000). This produces an amplitude modulation of their sinusoidal biases and allows us
Table 7.1. Two eight channel test configurations.

<table>
<thead>
<tr>
<th>component</th>
<th>Test A</th>
<th>Test B</th>
</tr>
</thead>
<tbody>
<tr>
<td>SQUID Controller GBWP</td>
<td>100 MHz</td>
<td>500 MHz</td>
</tr>
<tr>
<td>Inductor</td>
<td>20 $\mu$H (BK1)</td>
<td>16 $\mu$H (BK2)</td>
</tr>
<tr>
<td>Demodulator</td>
<td>SRS 830</td>
<td>custom electronics</td>
</tr>
</tbody>
</table>

Table 7.2. Frequency schedule for eight channel test.

<table>
<thead>
<tr>
<th>channel</th>
<th>frequency (kHz)</th>
<th>optical/dark</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>375</td>
<td>dark</td>
</tr>
<tr>
<td>2</td>
<td>425</td>
<td>optical</td>
</tr>
<tr>
<td>3</td>
<td>485</td>
<td>dark</td>
</tr>
<tr>
<td>4</td>
<td>555</td>
<td>dark</td>
</tr>
<tr>
<td>5</td>
<td>635</td>
<td>optical</td>
</tr>
<tr>
<td>6</td>
<td>725</td>
<td>dark</td>
</tr>
<tr>
<td>7</td>
<td>825</td>
<td>dark</td>
</tr>
<tr>
<td>8</td>
<td>945</td>
<td>calibration resistor</td>
</tr>
</tbody>
</table>

to quantify crosstalk between an optical channel and adjacent dark channels. Table 7.2 shows the designed frequency schedule for the eight channel tests (note: for Test A, we used a pair of 40 $\mu$H inductors in parallel to give us 20 $\mu$H.)

The sinusoidal voltage biases are produced by Direct Digital Synthesis (DDS) ICs (Analog Devices P/N AD9854). Each DDS produces a stable sinusoidal bias in the frequency range at which we bias our TES bolometers. The output current digital-to-analog converter (DAC) is 12-bit, the frequency tuning adjust is 48-bit and the phase adjust is 14-bit. Each DDS IC produces a quadrature output: the I-DAC and Q-DAC produce signals at identical frequencies and 90° out of phase.

The demodulator used for Test A was a Stanford Research Systems (SRS) single channel digital lock-in amplifier (model SR844). The SR844 has a bandwidth of 200 MHz, 16-bit input analog-to-digital converters (ADCs) and a minimum time constant of $\sim 100 \mu s$ which is sufficiently fast to measure bolometer time constants of 200 $\mu s$ to 1 ms.

For Test B, we used the production version of the custom demodulator developed for the FDM system. The core of the custom demodulator is a Tayloe switch [66]. The switch is implemented with a Texas Instruments FET-based bus switch IC (TI P/N 74CBT3253)
that is clocked with a pure unmodulated signal from the same DDS IC used to supply the sinusoidal bias. An active 8-pole Butterworth filter bandwidth limits the output of the Tayloe switch to 400 Hz [67].

7.2 Eight Channel Test: Performance

The eight channel tests are intended to be demonstrations of the feasibility of our FDM system. In particular, we verify the FDM performance by measuring four properties of the readout:

- 4K transfer function (verify center frequencies, Q of LC filters)
- Bolometer IV curves (measures the health of the bolometer under AC bias, multiplexed readout)
- Bolometer demodulated noise (measures the sensitivity of multiplexed bolometer)
- Crosstalk between adjacent detectors (must be less than 0.01)

7.2.1 Cold Transfer Function

The 4K transfer function measures the current flowing through the voltage-biased eight channel test as a function of bias frequency. From the transfer function, we verify the center frequencies and Q values of the LC filters. The center frequencies need to be measured to within 500 Hz so that we can deliver electrical bias power to every multiplexed detector in the set. The Q values are measured to ensure that the LC filter has the correct bandwidth and that there is no extra resistance in series with the bolometer.

Table 7.3 contains the results of the cold transfer function measurement. The resonance frequencies were lower than the designed values by between 7 and 10%. We interpret this as a small increase in the capacitor dielectric constant, $\epsilon$, as the capacitors cool from 300K to 300 mK. Since the spread in the change is close to 3%, this change can be controlled.
Table 7.3. Measured Frequencies (Test A)

<table>
<thead>
<tr>
<th>channel</th>
<th>designed frequency (kHz)</th>
<th>measured frequency (kHz)</th>
<th>Q value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>375</td>
<td>345.5</td>
<td>correct Q</td>
</tr>
<tr>
<td>2</td>
<td>425</td>
<td>396</td>
<td>correct Q</td>
</tr>
<tr>
<td>3</td>
<td>485</td>
<td>452.5</td>
<td>correct Q</td>
</tr>
<tr>
<td>4</td>
<td>555</td>
<td>503</td>
<td>correct Q</td>
</tr>
<tr>
<td>5</td>
<td>635</td>
<td>584.5</td>
<td>correct Q</td>
</tr>
<tr>
<td>6</td>
<td>725</td>
<td>671.5</td>
<td>correct Q</td>
</tr>
<tr>
<td>7</td>
<td>825</td>
<td>744.5</td>
<td>correct Q</td>
</tr>
<tr>
<td>8</td>
<td>945</td>
<td>N/A</td>
<td>low Q</td>
</tr>
</tbody>
</table>

by correcting the 300K capacitor values while preserving the designed frequency spacing between channels.

The measured Q values for all channels except for the calibration resistor match the designed values. The 0.5 Ω dummy resistor exhibited a low Q value because of a bad solder connection. This connection was repaired and in Test B the calibration resistor channel showed the correct Q value.

7.2.2 Bolometer IV Curves

We measure TES bolometer IV curves for multiplexed detectors to ensure that the detectors exhibit turn-arounds (i.e. descend into their superconducting transitions) and that in the transition, the electrical power dissipated approaches a constant value. Using this constant power, the operating temperature of the TES bolometer, $T_b$, and the bath temperature, $T_s$, we can measure the G of the sensor (see chapter 3).

Figure 7.1 shows the measured IV and PV (power vs. voltage) curves for channel 2 (396.0 kHz) using FDM readout. The IV curve shows a clean turn-around a voltage bias that indicates $G = 67$ pW/K and the PV curve shows that the TES bolometer operates in the expected constant power regime below its turn-around.
7.2.3 Bolometer Noise

As discussed in chapter 3, the bolometer sensitivity is expected to be limited by thermal fluctuations in the link between the thermistor at $T_b$ and the heat bath at $T_s$. The NEP for such a detector is thus expected to be $NEP = \gamma \sqrt{4kT_b^2G}$. Measurements of noise with single-channel readout of DC biased TES bolometers have yielded white noise levels (measured between 2 Hz and 100 Hz) that agree with this estimate of the NEP.

We measured the demodulated noise of multiplexed sensors to ensure that the sensitivity of sensors under FDM readout is not degraded. Figure 7.2 shows the demodulated noise spectrum, again of channel 2 in Test A, biased at a frequency of 396 kHz and an operating resistance of 0.6 $R_n$. 

Figure 7.1. IV and PV curves for channel 2 with FDM readout (Test A)
The spectrum is white at frequencies down to 200 mHz. The measured noise spectrum corresponds to a NEP of $2.5 \times 10^{-17} \text{ W}/\sqrt{\text{Hz}}$. By comparison, we predict a NEP $= (4k_B T^2 G)^{1/2} \approx 2.1 \times 10^{-17} \text{ W}/\sqrt{\text{Hz}}$ using our sensor parameters (we ignore the factor of $\gamma$, which should be very close to unity). Here, $G$ is the thermal conductance of the sensor to the bath, $6.7 \times 10^{-11} \text{ W}/\text{K}$. The readout noise contributes an NEP of $0.8 \times 10^{-17} \text{ W}/\sqrt{\text{Hz}}$ which, added in quadrature with the NEP prediction, brings the total estimated NEP to $\sim 2.2 \times 10^{-17} \text{ W}/\sqrt{\text{Hz}}$, in good agreement with the measured white noise.

### 7.2.4 Crosstalk

The optical crosstalk between adjacent millimeter-wave bolometers is typically 0.01. Thus, it is important that the crosstalk between multiplexed channels be less than this value. We expect two crosstalk mechanisms arising from the multiplexed readout. The finite bandwidth of the tuned filters allows currents to flow through a channel from off-
resonance carriers. The currents from the two adjacent channels, \( I_{i \pm 1} \), as a fraction of on-resonance bias \( I_i \) for channel \( i \), are 
\[
I_{i \pm 1}/I_i = R_i/\sqrt{R_i^2 + (\omega_{i \pm 1}L_i - 1/\omega_{i \pm 1}C_i)^2}
\]
As \( R_i \) changes in response to an optical signal, the off-resonance currents are modulated. However, at the adjacent frequencies \( \omega_{i \pm 1} \) the impedance of channel \( i \) is dominated by the tuned filter, not by \( R_i \), and the modulation of the off-resonance currents is very small. For our circuit parameters, the ratio of the off-resonance current modulation (from an adjacent channel) to the on-resonance current modulation, which we define as the crosstalk, is \( 4 \times 10^{-5} \).

The second crosstalk mechanism is inductive coupling between coils. Adjacent coils are coupled by a measured coefficient \( k = 0.010 \pm 0.002 \), yielding a mutual inductance \( M_{i,i \pm 1} = 0.4 \, \mu \text{H} \) (see chapter 6). When a current \( I_i \) flows through channel \( i \), voltages \( |V_{i \pm 1}| = \omega_i M I_i \) are induced in channels \( i \pm 1 \), causing off-resonance currents to flow. As the \( R_{i \pm 1} \) change due to optical signals, these currents are modulated. Again, the modulation is small since the impedances of channels \( i \pm 1 \) at the frequency of current \( I_i \) are dominated by the tuned filters, not \( R_{i \pm 1} \). For the stated mutual inductance, the ratio of the coupled current modulation to the on-resonance current modulation is \( 7 \times 10^{-5} \).

Figure 7.3. Demodulated spectra of an optical sensor and a dark sensor. A cold LED is illuminating the optical sensor with an 84 Hz signal.
We measure crosstalk between sensors at adjacent frequencies by modulating the cold LED at 84 Hz and monitoring the spectra of adjacent optical and dark sensors, demodulated with a lock-in amplifier. Figure 7.3 shows the demodulated spectra of an optical sensor biased at 396 kHz and a dark sensor biased at 452.5 kHz. Both sensors are biased at 0.45 $R_{ni}$. The absence of a 84 Hz signal in the dark sensor spectrum sets an upper limit of 0.004 on crosstalk between adjacent channels. This level of crosstalk is well within our design requirements.

7.2.5 Low Frequency Noise and Vibration

In the demodulated noise spectrum shown in figure 7.2, the noise was white at frequencies above 200 mHz. Below 200 mHz, however, the noise rises as the frequency decreases. Non-white low frequency noise can severely limit the science potential any mm-wave experiment. We would prefer that the sky signals containing cosmologically significant information be at frequencies where the detector noise is white and stationary in order to construct reliable models of the noise. Large amounts of low frequency noise require us to push telescope scan speeds higher in order to move sky signals higher in frequency. Scan speeds cannot be increased indefinitely, and so large amounts of low frequency noise will quickly reduce experimental sensitivity. Thus, there is a strong motivation for understanding and minimizing low frequency noise.

There are several candidate sources for low frequency noise. The most common sources of low frequency noise are:

- variations in detector bath temperature $T_s$
- variations in the amplitude of the bias and nulling carriers
- variations in RFI heating of bolometers
- intrinsic low frequency fluctuations in the detectors themselves

Variations in detector bath temperature can be measured either by directly measuring the slow changes in temperature via the thermometry attached to the $^3$He fridge or by
correlating signals simultaneously from a pair of multiplexed sensors. Both of these methods show that while there is some small temperature variation in the bath, this variation is too small to produce the low frequency noise typically seen in TES bolometers.

Taking a carrier and nuller signal from a DDS chip and measuring the low frequency variation of the difference in these signals shows a variation of $0.25 \times 10^{-5}$ of the carrier amplitude at frequencies near 1 Hz. The variation increases roughly as $1/f$ at frequencies lower than 1 Hz and has been traced to the current DAC (digital-to-analog converter) at the output of the DDS chip. This variation is large enough to explain the 200 mHz knee in figure 7.2. By using the same DDS chip to produce the carrier and nulling signal, we can push this variation down to $< 10^{-6}$ of the carrier amplitude at frequencies near 1 Hz.

Severe amounts of low frequency noise seem well correlated with poor rejection of radio-frequency interference (RFI) at the detectors. Figure 7.4 shows the difference between demodulated spectra taken while RFI open (improper shielding) and RFI closed (proper shielding). The striking increase in low frequency noise is most likely due to slow variations in RFI heating of the bolometer. The low frequency noise disappears when the TES bolometer is biased normal which sends its responsivity to absorbed power to zero.

Another striking feature of figure 7.4 is the presence of a 10 Hz peak in the demodulated spectrum when the cryostat is shaken at 10 Hz with a Ling driver (vibrations approximately 50-80 mg in amplitude). This peak is only present in the RFI open configuration. There is a strong connection between RFI and vibration sensitivity: by properly shielding the detectors from RFI, the response to vibration disappears. The connection is most likely due to the detector moving in a non-uniform RF environment: if the detector is physically translating in this RF environment at 10 Hz, then a 10 Hz power signal is being delivered to the detector. Removing the source of this power, the RFI, removes this source of vibration sensitivity.

In fact, we expect that sinusoidally biased low impedance TES bolometers will in general suffer less from heating due to vibrations that change the capacitance to ground than higher impedance and DC biased devices. Microphonic heating is less significant for low impedance
devices, and the tuned filters in series with our bolometers will impede the induced current flow at low frequencies.

When the detectors are properly shielded from RFI, we measure an extreme vibration insensitivity in our multiplexed sensors. We use an audio driver bolted to the side of our test dewar to produce vibrations from 10 Hz to 1 kHz. A cold piezoelectric film measures the vibration environment at the sensors. Figure 7.5 shows the demodulated spectrum of a sensor undergoing 10 Hz vibrations. There is no evidence for vibration sensitivity in the sensors’ demodulated spectra when the sensors experience accelerations of up to 50 mg from 10 Hz to 1 kHz.

The last potential source of low frequency noise is intrinsic fluctuations in the detectors themselves. In the FDM system tests, including the two eight channel tests, there is no evidence yet for this intrinsic low frequency noise.

Figure 7.4. RFI induced low frequency noise.
7.2.6 Summary

The results of all four of the tests outlined at the start of this section are satisfactory: predictable center frequencies, correct Q values and multiplexed detector sensitivities and sufficiently low crosstalk between channels adjacent in frequency all meet our requirements for fielding the eight channel FDM system in a mm-wave receiver.

In understanding low frequency noise, we have also shown that the FDM system is remarkably insensitive to mechanical vibrations. Although in hindsight this insensitivity is expected for a low impedance, LC filtered system, its is a very important benefit for systems that will be integrated with telescopes with harsh vibration environments.

The next section discusses some extrapolations based on the results of the eight channel test on the number of channels we can expect to multiplex with the current generation of FDM technology. Finally, the following chapter discusses exciting extensions to the current FDM technology.
7.3 Limits on Channel Count

There are two properties of the FDM system that determine the number of channels that can in principle be multiplexed: the channel spacing, $\zeta$ and the closed loop bandwidth, $B$. Given these two properties, the number of possible multiplexed channels is $n = \frac{B}{\zeta}$.

Dynamic Range

However, this calculation is too simple. It does not take into account the dynamic range required from the SQUID electronics to accommodate $n$ carriers. To estimate this quantity, along with the other calculations in this section, we assume the following system parameters:

Table 7.4. System Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td>350 pW/K</td>
</tr>
<tr>
<td>$T_s$</td>
<td>0.25 K</td>
</tr>
<tr>
<td>$T_b$</td>
<td>0.5 K</td>
</tr>
<tr>
<td>$P_{\text{radiation}}$</td>
<td>16 pW</td>
</tr>
<tr>
<td>$f$</td>
<td>150 GHz</td>
</tr>
<tr>
<td>$\Delta f$</td>
<td>38 GHz</td>
</tr>
<tr>
<td>$R_n$</td>
<td>1.25 \Omega</td>
</tr>
<tr>
<td>$R_{\text{operating}}$</td>
<td>0.5 \Omega</td>
</tr>
<tr>
<td>SQUID electronics noise</td>
<td>5 pA/\sqrt{Hz}</td>
</tr>
<tr>
<td>$L_{\text{tuning}}$</td>
<td>16 \mu H</td>
</tr>
</tbody>
</table>

This implies (see chapter 3):

- a carrier amplitude of $12 \mu A_{rms}$
- wideband Johnson noise of $\sqrt{4kT/R} = 7.4 \text{pA}/\sqrt{\text{Hz}}$
- sensor time constant limited thermal noise of $\sqrt{4kT^2G/V_{bias}} = 11.6 \text{pA}/\sqrt{\text{Hz}}$
- sensor time constant limited photon shot noise of $\sqrt{2h\nu P_{rad} + P_{rad}^2/(\Delta \nu)/V_{bias}} = 16.7 \text{pA}/\sqrt{\text{Hz}}$

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- a combined thermal and photon noise of 20.3 pA/√Hz (Johnson noise suppressed by electrothermal feedback)

With \( n \) carriers in place, the maximum current flowing through the SQUID electronics input is \( n \cdot 12 \mu A_{\text{rms}} \), or \( n \cdot 34 \mu A_{\text{pp}} \). That is, in order to accommodate these \( n \) carrier simultaneously \emph{without nulling}, the SQUID electronics requires a dynamic range of \( n \cdot 34 \mu A_{\text{pp}} \). Appendix A outlines the calculation of dynamic range for a given loop gain in the SQUID electronics. For a loop gain of 50, which is the current design of SQUID electronics, this limits \( n \) to between 12 and 13. However, if we relax the constraint that we need to measure all \( n \) detectors simultaneously without nulling, then \( n \) can increase by the nulling factor. We routinely null by close to a factor of 10 which means the channel count with the current generation of SQUID electronics can approach 100. Our limit on channel count then reverts back to \( n = \frac{\mathcal{G}}{\zeta} \).

\( \zeta: \text{Channel Spacing} \)

There are three things that limit the minimum channel spacing we choose for the FDM system. The first is that we require that the LC filter center frequencies are spaced far enough apart that the Johnson noise from the other channels does not degrade the sensitivity of a sensor. If we compare the amount of Johnson noise leakage from the other sensors to the SQUID electronics noise, then the Johnson noise only adds 2\% extra noise with a 30 kHz channel spacing. This requirement does not produce a strong limit on the channel spacing.

The second consideration is the phase shift produced by parasitic currents flowing through the other \( n - 1 \) channels when tuned to a particular channel. This phase shift is the largest for the lowest and highest frequency channels. We require that phase shifts be less than 22° in order to null TES bolometer carrier effectively without adding low frequency noise. Table 7.5 shows the maximum negative and positive phase shifts for different channel spacings. To meet our requirement, we require that \( \zeta > 50-60 \text{ kHz} \).

Finally, we want to pick a \( \zeta \) that keeps the crosstalk between channels adjacent in
Table 7.5. Phase shifts induced by parasitic current flow through other channels.

<table>
<thead>
<tr>
<th>spacing (kHz)</th>
<th>negative phase shift °</th>
<th>positive phase shift °</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>-26</td>
<td>21</td>
</tr>
<tr>
<td>40</td>
<td>-24</td>
<td>18</td>
</tr>
<tr>
<td>45</td>
<td>-21</td>
<td>16</td>
</tr>
<tr>
<td>50</td>
<td>-19</td>
<td>14</td>
</tr>
<tr>
<td>55</td>
<td>-18</td>
<td>13</td>
</tr>
<tr>
<td>60</td>
<td>-17</td>
<td>12</td>
</tr>
<tr>
<td>65</td>
<td>-16</td>
<td>11</td>
</tr>
<tr>
<td>70</td>
<td>-15</td>
<td>10</td>
</tr>
</tbody>
</table>

frequency below 1%. The calculation outlined in the previous section can be carried out for several channel spacings. Spacings above $\zeta = 40$ kHz keep the crosstalk to below 1%.

The firm limit on channel spacing seems to be 50 kHz. Decreasing below this spacing produces large carrier phase shifts and increasing crosstalk between adjacent channels.

B: FLL Bandwidth

The bandwidth of the SQUID electronics is currently 1.1 MHz and is limited by the wire lengths between 300K and 4K. This bandwidth could be increased at the expense of closed loop gain, but dynamic range and linearity of the FLL is then sacrificed.

One of the most promising ways of increasing the bandwidth of the current generation of SQUID electronics is to alter the component values of the lead-lag filter, described in chapter 5 and appendix B. The lead-lag filter exists to preserve the FDM system bandwidth while simultaneously decreasing the FLL gain at high frequencies (> 5 MHz). By using smaller capacitors, $C_s$, we can increase the FDM bandwidth but retain the gain reduction at high frequencies and thus the wiring length constraints on the FLL bandwidth. Full circuit simulations indicate that stable FLL performance with larger bandwidths can be achieved with this strategy.

By increasing the GBWP of the first stage amplifier to between 1.5 and 2.0 GHz and decreasing $C_s$ to 350 pF, we can easily double the bandwidth of the FLL electronics without sacrificing phase margin, provided that we maintain the 0.2 m wire length limit between
Table 7.6. Stable FLL Bandwidths for different lead-lag filter configurations

<table>
<thead>
<tr>
<th>$C_s$</th>
<th>GBWP of 1st stage</th>
<th>-3dB frequency</th>
<th>Unity gain frequency</th>
<th>FLL Phase Margin</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 nF</td>
<td>1.5 GHz</td>
<td>1.1 MHz</td>
<td>20.4 MHz</td>
<td>60°</td>
</tr>
<tr>
<td>1 nF</td>
<td>2.0 GHz</td>
<td>1.2 MHz</td>
<td>23.4 MHz</td>
<td>60°</td>
</tr>
<tr>
<td>500 pF</td>
<td>1.5 GHz</td>
<td>1.7 MHz</td>
<td>23.8 MHz</td>
<td>45°</td>
</tr>
<tr>
<td>500 pF</td>
<td>2.0 GHz</td>
<td>1.9 MHz</td>
<td>26.7 MHz</td>
<td>45°</td>
</tr>
<tr>
<td>350 pF</td>
<td>1.5 GHz</td>
<td>2.0 MHz</td>
<td>26.7 MHz</td>
<td>36°</td>
</tr>
<tr>
<td>350 pF</td>
<td>2.0 GHz</td>
<td>2.4 MHz</td>
<td>29.6 MHz</td>
<td>36°</td>
</tr>
<tr>
<td>250 pF</td>
<td>1.5 GHz</td>
<td>2.2 MHz</td>
<td>30.1 MHz</td>
<td>29°</td>
</tr>
<tr>
<td>250 pF</td>
<td>2.0 GHz</td>
<td>2.7 MHz</td>
<td>33.3 MHz</td>
<td>29°</td>
</tr>
<tr>
<td>125 pF</td>
<td>2.0 GHz</td>
<td>2.9 MHz</td>
<td>44.8 MHz</td>
<td>10°</td>
</tr>
</tbody>
</table>

300K and 4K. Pushing the bandwidth much higher than this decreases the phase margin unacceptably.

Tests with actual LC filters attached to the SQUID electronics must be performed to verify that this doubling of bandwidth does indeed maintain stable FLL performance as the full circuit simulations suggest.

With MUX frequencies between 500 kHz and 2 MHz with 50 kHz spacing we can increase the FDM channel count to 30. If we stay within an octave (1 MHz to 2 MHz), the FDM channel count is 20. In order to approach channel counts of 100, a new generation of SQUID electronics with a factor of 5 to 10 increase in bandwidth is needed.
Chapter 8

LISA

We have developed and demonstrated a first generation eight-channel FDM system that is ready for integration with a large-format array of TES bolometers for mm-wave observations. Chapter 7 discussed the eight channel test of this system. We concluded that to increase the number of channels being multiplexed to 100 we need a large increase in the FLL bandwidth. The current generation of SQUID electronics has a FLL bandwidth of 1 MHz. Although we have discussed schemes for increasing this to 1.5 to 2 MHz, the bandwidth is ultimately limited by the gain stage at room temperature and thus by the length of wiring connecting the 300K and 4K components of the SQUID electronics.

A promising strategy for increasing the bandwidth of the SQUID electronics eliminates this room temperature gain stage. The primary feedback path moves to 4K and the wire lengths involved shrink from 0.2 m to 0.02 m. In this chapter we discuss the cold feedback scheme that uses a SQUID opamp: the LISA - Linearized SQUID Array. We discuss the requirements for the LISA and present the performance of a prototype LISA.

8.1 Introduction

The LISA is designed to have the necessary FLL gain after removing the room temperature amplifier from the primary feedback loop. An entirely cold feedback configuration has
already been developed, the ”SQUID op-amp” [68]. This configuration relies on cascading several SQUIDs to achieve high loop gain and linearity. However, the configuration does not achieve the extension in dynamic range necessary for the FDM system.

The LISA is a series array of many SQUIDs. For the prototype we use three 100-element SSAs in series to produce an effective 300-element SSA. We feed the output voltage, $V_{out}$, through a feedback resistor $R_{fb}$ directly to the input coils of the SSAs. Figure 8.1 shows a schematic of the LISA.

![Figure 8.1. Schematic of 3-SSA LISA](image)

The requirements of the LISA are similar to the requirements listed in chapter 5 for the SQUID electronics. We assume a TES bolometer with an operating impedance of 0.5 Ω, white noise of 10-20 pA/√Hz, and a bias current that does not exceed 12 µA$_{rms}$.

- **dynamic range:** the LISA must be able to measure a single carrier ($\leq 12$ µA$_{rms}$ OR $\leq 34$ µA$_{pp}$) without nulling. In practice, with the NIST 8-turn SQUIDs that we use (26 µA/Φ$_0$), this means a flux-locked loop gain of at least 3 to 4.

- **bandwidth:** the bandwidth of the LISA must exceed the bandwidth limitation of the current FDM system of 1 MHz by a factor of at least 5 to 10.
- **low current noise**: the LISA input current noise should be well below the bolometer white noise level, i.e. $< 5 \text{ pA}/\sqrt{\text{Hz}}$.

- **high trans-impedance**: the LISA transimpedance needs to be high enough so that the current noise floor of the bolometers ($\sim 10 \text{ pA}/\sqrt{\text{Hz}}$) is converted into a voltage noise that dominates the $1\text{nV}/\sqrt{\text{Hz}}$ at the input of the first stage of warm electronics that follow the LISA. In practice this means $Z_{\text{LISA}} \geq 150\Omega$.

- **linearity**: the LISA must have sufficient linearity across this dynamic range to prevent the distortion signals in a comb of carriers from corrupting the information carried on each carrier.

The performance of the LISA depends on the closed loop gain of the feedback loop. For the four-component circuit (3 SSAs in series, single feedback resistor) shown in figure 8.1, the loop gain is

$$A_{\text{LISA}} = \frac{Z_{\text{sq}}}{R_{\text{sq}} + R_{\text{sq}}}$$

where $R_{\text{fb}}$ is the feedback resistor, $R_{\text{sq}}$ is the output impedance of the 3 SSAs in series (assumed to be completely resistive), and $Z_{\text{sq}}$ is the transresistance of the 3 SSAs in series.

The loop gain produces the increase in dynamic range through feedback. For small signals at the input of the LISA, the net current flowing through its input coil is:

$$i_{\text{sq}} = \frac{i_{\text{in}}}{1 + A_{\text{LISA}}}$$

The forward-gain of the LISA (measured in $\Omega$s and called the transimpedance of the LISA) also depends on $R_{\text{fb}}$, $R_{\text{sq}}$, and $Z_{\text{sq}}$:

$$\frac{v_{\text{out}}}{i_{\text{in}}} = Z_{\text{LISA}} = [(R_{\text{fb}} + R_{\text{sq}}) \parallel Z_{\text{sq}}] \left(1 - \frac{R_{\text{sq}}}{R_{\text{fb}} + R_{\text{sq}}}ight)$$

From equations 8.1 and 8.3, we see that there are two conflicting constraints on choosing $R_{\text{fb}}$: to achieve higher loop gains, we choose $R_{\text{fb}}$ as small as possible to maximize $A_{\text{loop}}$ in
equation 8.1. As $R_{fb}$ approaches zero, we approach the limit of loop gain we can achieve, $A_{MAX} = \frac{Z_{sq}}{R_{sq}}$.

This limit depends only on properties of the individual SQUIDs that are placed in series to build the LISA. We can see that to get the best performance with the LISA, we need to choose SQUIDs that maximize the ratio $A_{MAX}$.

The problem with minimizing $R_{fb}$ is that as we decrease $R_{fb}$, we also decrease $Z_{LISA}$. To maintain a large forward gain, or LISA transimpedance, we want to increase $R_{fb}$. The right strategy then for choosing $R_{fb}$ is to choose a $Z_{LISA}$ that amplifies the $10 - 20$ pA/$\sqrt{\text{Hz}}$ expected from the bolometer to well above the input noise voltage of the first room temperature stage, which is expected to be close to $1$ nV/$\sqrt{\text{Hz}}$.

The 100-element SSAs used to develop the LISA have $R_{sq} \sim 85$ Ω and $Z_{sq} \sim 400$ Ω. $Z_{LISA} > 150 - 200$ Ω satisfies the requirement of overriding the room temperature stage noise floor which, assuming three SSAs in series, requires $R_{fb} > 200$ Ω. For the LISA prototype, we use $R_{fb} = 200$ Ω with three 100-element NIST 8-turn SSAs in series.

8.2 LISA Dynamic Range

8.2.1 Dynamic Range Measurement

Figure 8.2 shows the open loop response of the 3-SQUID LISA. These data were taken with a lead-lag capacitor ($C_p = 470$ pF) in place to bandwidth-limit the LISA to < 6 MHz. As expected, the outputs of all three SQUID arrays are adding coherently (each SQUID array has between 4.3 and 4.5 mV$_{pp}$ output). The overall maximum output is 13.8 mV$_{pp}$, a value completely consistent with adding the three individual SQUID array outputs coherently.

Figure 8.3 shows the same open loop response along with the response of the LISA with the cold feedback loop closed (with a 200 Ω resistor). The LISA response has a "linear" regime within the dynamic range of the cold flux-locked loop (from -20 μA to 20 μA) and a "sawtooth" regime at higher applied currents (above and below the linear regime).
We have a dynamic range of approximate 40\(\mu A_{pp}\). We expect that our open loop LISA transresistance is close to 1500 \(\Omega\). This, combined with a feedback resistance of 230 \(\Omega\) and a output resistance of 240 \(\Omega\), gives us a loop gain of 3.4. This loop gain, using the equation derived in appendix A

\[
i_{max,pp} = 26 \mu A \left(\frac{1}{\pi} + \frac{A_{loop}}{2\pi}\right) \tag{8.4}
\]

predicts a dynamic range of 42 \(\mu A_{pp}\), very close to the dynamic range we observe.

### 8.2.2 LISA Tuning

Tuning the LISA with the cold feedback loop connected is straightforward. To determine the optimum SQUID bias current, several of these voltage-flux curves can be measured for different SQUID biases. The SQUID bias producing the maximum dynamic range (largest current range in the "linear" regime) is then chosen.

The flux bias tuning is even more straightforward. Once the voltage-flux curve is mea-
Figure 8.3. Open and cold closed loop response (voltage to flux characteristic) of a 3-SQUID LISA.

sured for the appropriate SQUID bias, the flux is adjusted to so that the LISA is operating in the middle of the "linear" regime giving it the maximum dynamic range.

### 8.2.3 Secondary Feedback Loop

We can increase the dynamic range of the LISA, for a given cold feedback resistor, by closing a second flux-locked loop containing the warm first stage amplifier. Figure 8.4 compares the voltage-flux curve of the LISA with only cold feedback and with cold feedback plus a secondary flux locked loop.

All of the above data were taken, as mentioned above, with the lead-lag filter in place with a capacitance of 470 pF. When the LISA bandwidth is increased by lowering this capacitance, the performance of the flux-locked loop begins to degrade. Table 8.1 shows the data along with the expected values of the dynamic range for each feedback topology.

Notice that as the lead-lag capacitance decreases, the dynamic range for a given feedback
Figure 8.4. Cold closed loop response (voltage to flux characteristic) of a 3-SQUID LISA. The black curve shows the voltage-flux with just the cold feedback in place, and the blue curve shows voltage-flux curve with the secondary loop closed with a 3.3 kΩ feedback resistor.

topology tends to decrease. For the 200 pF and 100 pF capacitors, the dynamic range cannot even be measured: the voltage-flux curve becomes severely distorted.

The reduction in the dynamic range and the severe distortion in several of the voltage-flux curves is strong evidence for a flux-locked loop instability and a high-frequency oscillation in the LISA. We also notice a DC jump on locking the flux-locked loop for the feedback topologies that produce this distortion, supporting the instability hypothesis. Further evidence for this hypothesis will be discussed in the LISA bandwidth section of this memo.

Finally, notice in table 8.1 that while the measured and expected dynamic ranges match well for cold feedback only and for cold feedback and a 10 kΩ secondary loop, the agreement is poorer for the lower feedback resistors. The calculations all assume a pure sinusoidal response for the LISA. This assumption might produce some errors in the calculated LISA response for large loop gains.
Table 8.1. Dynamic range measurements for the 3-SQUID LISA

<table>
<thead>
<tr>
<th>lead-lag cap (pF)</th>
<th>dynamic range µA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no sec. loop</td>
</tr>
<tr>
<td>470</td>
<td>38.7</td>
</tr>
<tr>
<td>300</td>
<td>36.5</td>
</tr>
<tr>
<td>200</td>
<td>36.1</td>
</tr>
<tr>
<td>100</td>
<td>30.8</td>
</tr>
<tr>
<td>300 (directly across 1st stage)</td>
<td>37.0</td>
</tr>
<tr>
<td>calculated</td>
<td>39.4</td>
</tr>
</tbody>
</table>

8.3 LISA Bandwidth

8.3.1 Primary Loop Only

I measured the forward-gain bandwidth by applying a small signal through the nulling input of the SQUID controller with the HP4195 and measuring the output voltage of the SQUID controller with the HP4195. Figure 8.5 shows the measured transfer functions (calibrated to ohms) for the different lead-lag capacitor values discussed in the previous section.

We are expecting, given both the LISA parameters and the results from the dynamic range section, a forward gain of 172 Ω compared to the measured value of 200 Ω. However, the gain calibration of the HP4195 network analysis is only good to within about 1 dB, which could explain this discrepancy. The bandwidths increase with decreasing capacitance as expected. The forward gain shows a peak at 43 MHz and a sharp drop off above this frequency for all lead-lag configurations. Spice simulations of this circuit configuration, assuming 10 Ω twisted pair from room temperature to 4 K with 2 ns delay, are shown in figure 8.6. Notice qualitatively similar behaviour with a good match at lower frequencies, and a poor match at higher frequencies (the peak and sharp drop are occurring at a factor of several higher in frequency).

From the simulations, I estimate what the expected −3dB frequency should be for the different lead-lag configurations and compare these to the measured frequencies in table 8.2.
Table 8.2. Comparison of measured and expected forward-gain bandwidths for the LISA

<table>
<thead>
<tr>
<th></th>
<th>measured (MHz)</th>
<th>expected (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>470pF lead-lag</td>
<td>5.0</td>
<td>6.6</td>
</tr>
<tr>
<td>300pF lead-lag</td>
<td>10.9</td>
<td>8.9</td>
</tr>
<tr>
<td>200pF lead-lag</td>
<td>14.0</td>
<td>11.0</td>
</tr>
<tr>
<td>300pF no lead-lag</td>
<td>13.9</td>
<td>15.7</td>
</tr>
</tbody>
</table>

There is 15–20% agreement between the measured and expected bandwidths. The best model for the discrepancy between simulation and measurement is that the LISA bandwidth is being limited by the SSAs themselves. This seems to be supported by the measurements of phase shifts at high frequencies.

Figure 8.5 shows the measured forward-gain bandwidth of 3-SQUID LISA for several lead-lag configurations.

Figure 8.7 shows the measured and figure 8.8 shows the simulated phase shifts for the forward-gain of the LISA. Figure 8.9 shows a comparison between the measurements and simulation for the configuration where a 300pF capacitor was placed across the first stage input of the amplifier. The phase lag is increasing much more rapidly in the measurements.
compared to the simulations. I do not have a good explanation for this, but it does support the hypothesis that the flux-locked loop is unstable for large loop gains.

### 8.3.2 10K secondary loop

Figure 8.10 shows the measured forward gain with the LISA locked with a secondary loop (10 kΩ resistor) for the same lead-lag configurations. The most striking feature of the response is the peaks between 10 and 40 MHz. The simulations suggest that the peaks are caused by the presence of the unterminated transmission lines. Although I can re-create the peaks, especially for the 300 pF capacitor directly at the input of the amplifier, the low frequency of the measured peaks requires much larger time delays than are physical. It’s possible that there is another model that is producing the observed peaks.
8.4 LISA Noise

I used the HP4195 to measure the output noise of the LISA. Figure 8.11 shows the measurement for the LISA fully biased and the LISA turned off (no current bias to the SQUID arrays). These data were taken with a 300pF capacitance in the lead-lag filter. There is a slight increase in the measured noise with the LISA biased (average is 1.65 nV/rthz) compared to the noise with the LISA un-biased (average is 1.3 nV/rthz). I expect the noise to be dominated by the input noise of the OPA847 (1.0 nV/rthz). There is potentially an overall gain calibration error, but it is clear that there is a 25% increase in the noise with the LISA biased compared with the LISA under no current bias. The noise could be a combination of SQUID flux noise and Johnson noise from the 200 Ω cold feedback resistor.

The cold feedback resistor should be contributing $\sqrt{4k_b T_{LISA} R_{fb1}}$ which is 0.22 nV/√Hz. Individual SQUID arrays have an measured input current noise of $\sim 2.3 \text{ pA/}\sqrt{\text{Hz}}$. N SQUID
arrays in series have a current noise that is a factor of $\sqrt{N}$ higher. Taking these two noise sources into account produces very close to a 25% increase in the noise referred to the first stage amplifier input.

Figure 8.12 shows the measured noise for three different flux-locked loop configurations: cold feedback only, a 10 $k\Omega$ feedback resistor, and a 5 $k\Omega$ feedback resistor. The noise decreases for the secondary loops because I have not calibrated out the transimpedances of these configurations, I have only divided out by the gain of the SQUID controller: this is why the noise level appears to be decreasing with lower feedback resistances.

### 8.5 LISA Linearity

A very rudimentary measurement of linearity was performed by using the Boonton signal generator to input a 1 MHz signal to the input coil of the LISA and measuring the fundamental and first harmonic with at the output of the LISA with the HP4195. Table 8.3 shows the results. As expected, the linearity improves with both increasing loop gain.
Figure 8.9. Comparison between measured and simulated forward-gain phase shift of 3-SQUID LISA.

<table>
<thead>
<tr>
<th>configuration</th>
<th>output voltage (mV)</th>
<th>input current ($\mu A_{rms}$)</th>
<th>level of second harmonic (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>open</td>
<td>69</td>
<td>6.3</td>
<td>46.6</td>
</tr>
<tr>
<td>10K</td>
<td>43</td>
<td>6.4</td>
<td>57</td>
</tr>
<tr>
<td>open</td>
<td>32</td>
<td>2.9</td>
<td>53.1</td>
</tr>
<tr>
<td>10K</td>
<td>19</td>
<td>2.9</td>
<td>60.8</td>
</tr>
</tbody>
</table>

(open loop to 10K secondary loop) and decreasing input signal size. Further measurements and simulations need to be performed to verify that the linearity of the LISA matches expectations.

### 8.6 Conclusions

All the dynamic range, noise, and bandwidth measurements agree fairly well with the calculations and simulations. The only discrepancies are deviations from the expected forward-gain bandwidth as this bandwidth is increased. This can be attributed to an incomplete circuit simulation that overestimates the achievable bandwidth or a bandwidth limitation from the SSAs that comprise the LISA.
Figure 8.10. Measured forward-gain bandwidth of 3-SQUID LISA locked with a secondary loop (10 kΩ resistor) for several lead-lag configurations.

The measured phase shift at 10 MHz is also much larger than the simulations predict. This could indicate that we are approaching the highest frequency at which we can operate 3 SSAs in series. This phase shift is the main reason why we need to bandwidth-limit the LISA with a warm capacitance at the output to get the expected dynamic ranges. Unfortunately, this capacitance greatly reduces the bandwidth of the loop-gain. Adding a compensation resistor across the input coil of the LISA will also bandwidth-limit the LISA and is something that should be investigated.

Testing of the prototype LISA shows that it has the potential to replace the existing SQUID electronics: it has the noise and dynamic range performance we require and the bandwidth is already many times larger than the current SQUID electronics. We need to determine whether the existing bandwidth limits are fundamental to the SSAs in order to extend the performance of the LISA above 9 MHz.
Figure 8.11. Output noise of the LISA, referred to the input of the first stage amplifier (i.e. SQUID controller gain divided out). The black data is measured with the LISA biased and the blue data is measured with the LISA turned off.

Figure 8.12. Comparison of the output noise of the LISA between the cold feedback loop closed, closing the warm secondary loop with a 10 kΩ resistor and closing the warm secondary loop with a 5 kΩ resistor.
Chapter 9

Conclusion

We have demonstrated a promising multiplexing technology for TES bolometers, the frequency domain readout multiplexer. Our eight-channel demonstration shows that sensors multiplexed with the FDM system have identical sensitivities to non-multiplexed sensors and that crosstalk between sensors adjacent in frequency space is well below our design requirements of $\sim 1\%$. The sinusoidal bias at the core of this readout technology provides immunity from gain variations in the SQUID electronics: we have seen white noise in the demodulated spectra of multiplexed sensors down to 200 mHz. Moreover, we have found that TES bolometers readout with the FDM system are remarkably insensitive to mechanical vibrations. Vibrations as high as 50 mg at the sensor stage, which easily exceeds the expected vibration level from mechanical refrigerators or telescope motion, do not produce a discernible signal in the sensors. Finally, our FDM system provides zero power dissipation at the coldest temperatures ($< 1$ K), a clear advantage over a TDM system which must include dissipative elements at these temperatures.

The FDM system is now moving from the demonstration stage to the deployment stage. Several mm-wave astronomy instruments will use this technology to readout large arrays of TES bolometers. The APEX-SZ experiment, an SZ survey instrument, has a planned 330 TES bolometers readout with an eight-channel FDM system and is currently completing integration at UC Berkeley [69]. The South Pole Telescope, another SZ survey instrument,
has a planned 1000 TES bolometers and will be installed at the South Pole in early 2007 [3].
Finally, two CMB polarization anisotropy receivers, PolarBear and EBEX, both of which will deploy in the next two years, will use the FDM system.

Several extensions to the demonstrated FDM system are being pursued. To increase the channel beyond 8-30 sensors, a large increase in FLL bandwidth is needed. The LISA, described in chapter 8, is a clear strategy for increasing this bandwidth. The prototype LISA has promising performance, including an increase in bandwidth by a factor of 4-5 over the existing SQUID electronics. The next step will be to integrate the prototype LISA with TES bolometers and probe the ultimate bandwidth that can be achieved with this system. One possible limitation to scaling up to 100 channels in an FDM system will be the parasitic resonances of the tuning inductors, discussed in chapter 6 which will destabilize a high bandwidth LISA system. This should be tested, along with a test of washer-less tuning inductors, which will push this parasitic resonance much higher in frequency. Finally, McGill University is collaborating with us in developing a completely digital FDM system. The digital FDM system allows the 300 K power consumption to shrink by a factor of 10 and allows the system to be more easily extended to large channel counts. A prototype version of the digital FDM system, which includes carrier “comb” generation and demodulation completely in the digital domain, has been tested with TES bolometers and has achieved bolometer limited sensitivities.
Bibliography


[56] SEIKO Corporation.

[57] Inc. HYPRES.

[58] National Institute of Standards and Technology.


Appendix A

Shunt Feedback SQUID Electronics

This appendix outlines the differences between shunt and series feedback electronics and discusses the general properties of shunt feedback. The SQUID electronics are then described in detail, including derivations for several key equations that govern the behaviour of the SQUID electronics. Most of these derivations are from memos written by H.G. Spieler.

A.1 Series and Shunt Feedback - Input Impedance

There are several basic feedback circuit topologies. The two topologies that couple an input signal (voltage or current) to an amplified voltage output at the output are called series and shunt feedback. In the series feedback topology, the feedback network is effectively placed in series with the input of the amplifying element and in the shunt feedback topology, the feedback network is effectively placed in parallel with the input of the amplifying element. The fundamental way in which these topologies differ is in how the input impedance of the circuit, $Z_{in}$, is transformed by the feedback: with the series topology, the feedback increases the effective input impedance and with the shunt topology, the feedback decreases the effective input impedance [70].

A.1.1 Series Feedback

Figure A.1 shows a basic circuit schematic that illustrates the series feedback topology. The active element is simply a voltage amplifier with an open-loop voltage gain of $A_v$ (which may be frequency dependent; open-loop voltage gain refers to the ratio of output voltage to input voltage without any feedback applied back to the input; we assume that the gain is a negative ratio) and a general input impedance of $Z_{in}$.

To derive the effective input impedance of the circuit in figure A.1, consider the current flowing through the input of the amplifier, $i_{in}$:

$$i_{in} = \frac{v_{in} - v^r}{Z_{in}}$$  \hspace{1cm} (A.1)
where $v^*$ is the voltage across the resistor $R_g$. The current flowing across $R_f$ is given by:

$$i_f = \frac{v_{out} - v^*}{R_f}$$  \hspace{1cm} (A.2)

and finally, $v_{out} = A_v(v_{in} - v^*)$. Solving for the ratio of $v_{in}/i_{in}$ (which is just $Z_{in}$ for no feedback), we get the effective input impedance:

$$Z_T = Z_{in} \left( 1 + A_v \frac{R_g}{R_g + R_f} \right)$$  \hspace{1cm} (A.3)

The actual current flowing through the input impedance $Z_{in}$ has been reduced by the series feedback and thus the effective input impedance has been increased by the feedback circuit.

### A.1.2 Shunt Feedback

Figure A.2 shows a basic circuit schematic that illustrates the shunt feedback topology. As in the previous section, the active element is a voltage amplifier with a possibly frequency dependent gain, $A_v$ and a general input impedance $Z_{in}$.

We derive the effective input impedance of the circuit in figure A.2 by following the
input current $i_{in}$, which divides into a current flowing through the feedback impedance $Z_f$ and a current into the amplifier, that is, $i_{in} = i_f + i_a$. The feedback current is then

$$i_f = \frac{v_o - v_{in}}{Z_f} \quad (A.4)$$

The output voltage is $v_0 = A_v v_{in}$ and the amplifier current is $i_a = v_{in}/Z_{in}$. Solving for the ratio $v_{in}/i_{in}$ (which is just $Z_{in}$ for no feedback), we get the effective input impedance:

$$Z_T = Z_{in} \parallel \frac{Z_f}{1 - A_v} \quad (A.5)$$

The input current divides between the amplifier input and the feedback impedance. Thus, the input impedance $Z_{in}$ is in parallel with the active impedance $Z_f/(1 - A_v)$ and the effective input impedance is reduced.

### A.2 SQUID Electronics

Because of the need for a low input impedance, the SQUID feedback circuit uses the shunt topology. To use the result from the previous section, we need to define the open-loop voltage gain of the SQUID controller shown in figure A.3. There are two gain stages. A current flowing through the input coil of the SQUID produces an output voltage:
Figure A.3. Schematic showing gain stages of SQUID electronics.

\[ v_{sq} = i_{in} M_i V_\phi = \frac{v_{in} M_i V_\phi}{i \omega L_{in}} \]  

(A.6)

where \( M_i \) is the mutual inductance between the input coil and SQUID washer and \( V_\phi = \frac{\partial V}{\partial \phi} \). The room temperature gain stage has a frequency dependent gain (most conveniently modelled as a single pole roll-off): \( A_{va}(f) = A_{va}/(1 + i \omega \tau) \). The forward voltage gain of the SQUID electronics shown in A.3 is thus:

\[ A_v = -i M_i V_\phi \frac{A_{va}}{\omega L_i} \frac{A_{va}}{1 + i \omega \tau} \]  

(A.7)

### A.2.1 Input Impedance

From equation A.5, we see that the input impedance of the SQUID electronics is the parallel combination of the input inductance, \( i \omega L_i \) and the active impedance \( Z_f/(1 - A_v) \).

\[ \frac{Z_f}{1 - A_v} = \frac{Z_f}{1 + i \frac{M_i V_\phi}{\omega L_i} \frac{A_{va}}{1 + i \omega \tau}} = \frac{Z_f}{1 + i \alpha \frac{A_{va}}{1 + i \omega \tau}} \]  

(A.8)

where \( \alpha = M_i V_\phi/\omega L_i \) is the ratio of SQUID transimpedance to open-loop input impedance. Note that equation A.8 is simply the feedback impedance in parallel with the feedback impedance reduced by the forward gain of the SQUID electronics.
\[
\frac{Z_f}{1 - A_v} = Z_f \parallel \frac{Z_f}{i\alpha A_v/(1 + i\omega \tau)}
\] (A.9)

Everything needed to calculate the input impedance of the shunt feedback SQUID electronics is contained in equation A.9. Moreover, the derivation is valid for a general feedback network \(Z_f\). However, two limiting cases for real \(Z_f\) are of interest: when \(\omega \tau \ll 1\), the active impedance approaches:

\[
\frac{Z_f}{1 - A_v} = -i \frac{Z_f}{\alpha A_v}
\] (A.10)

For real feedback impedances, this is a capacitive impedance that approaches zero at low frequencies, completely shunting the input inductance. When \(\omega \tau \gg 1\), \(A_v\) begins to decrease and at high frequencies:

\[
\frac{Z_f}{1 - A_v} \approx Z_f
\] (A.11)

### A.2.2 Forward Gain - Transimpedance

A similar analysis can be performed to calculate the forward gain, or transimpedance, of the SQUID electronics. The transimpedance, \(Z_{\text{forward}}\) is defined as the ratio of output voltage to input current, \(Z_{\text{forward}} = v_{\text{out}}/i_{\text{in}}\). The input current splits into two currents, that flowing through the SQUID input coil and that flowing through the feedback impedance, \(i_{\text{in}} = i_{\text{sq}} + i_{\text{fb}}\). Calculating both of these yields:

\[
i_{\text{in}} = \frac{v_{\text{in}} - v_{\text{out}}}{Z_f} = \frac{v_{\text{out}}/A_v - v_{\text{out}}}{Z_f}
\]
\[
i_{\text{sq}} = \frac{v_{\text{in}}}{i\omega L_i} = \frac{v_{\text{out}}}{i\omega L_i A_v}
\]

Adding these produces:

\[
i_{\text{in}} = i_{\text{fb}} + i_{\text{sq}} = \frac{v_{\text{out}}}{Z_f} (1/A_v - 1) + \frac{v_{\text{out}}}{i\omega L_i A_v}
\] (A.12)

and simplifying:

\[
Z_{\text{forward}} = \frac{v_{\text{out}}}{i_{\text{in}}} = \frac{Z_f}{1/A_v - 1 + Z_f/(A_v i\omega L_i)}
\] (A.13)

Again, equation A.13 contains everything needed to calculate the transimpedance of the SQUID electronics. However, for most cases, when \(A_v \gg 1\), the forward gain approaches an extremely simple form:

\[
Z_{\text{forward}} \approx -Z_f
\] (A.14)
A.2.3 Dynamic Range

The last derivation for the SQUID electronics is quantifying the dynamic range extension of the SQUID electronics due to the shunt feedback. Again, we split the input current between the input coil and feedback impedance:

\[ i_{in} = i_{fb} + i_{sq} = \frac{v_i - A_v v_i}{Z_f} + i_{sq} \]  
(A.15)

The input voltage is related to the current through the input coil: \( v_{in} = i_{sq} \omega L_i \), and after expanding \( A_v \) we have:

\[ i_{in} = i_{sq} \left( 1 - \frac{A_v M V_\phi}{Z_f (1 + i \omega \tau)} + \frac{\omega L_i}{Z_f} \right) \]  
(A.16)

We can define a closed loop gain \( A_{loop} = \frac{A_v M V_\phi}{Z_f (1 + i \omega \tau)} \). Then for small signals at the input of the SQUID electronics (making the approximation \( \omega L_i \ll Z_f \)),

\[ i_{sq} = \frac{i_{in}}{1 - A_{loop}} \]  
(A.17)

Thus, the closed loop gain gives us the means to calculate the extension in dynamic range because of the shunt feedback. For \( A_{loop} \gg 1 \), the current through the input coil of the SQUID is reduced by a factor of \( A_{loop} \).

Notice that the definition of \( A_{loop} \) contains \( V_\phi \) which is in general a function of the current flowing through the SQUID. To correct for the change of \( V_\phi \) with applied current we follow [71] and use the large signal loop gain which is smaller by \( 2/\pi \):

\[ i_{sq} = \frac{i_{in}}{1 - 2 A_{loop} / \pi} \]  
(A.18)

The maximum \( i_{sq} \) that can be sustained is that which produces \( 1/4 \Phi_0 \). Thus, for the NIST 8-turn SSAs which need \( 26 \mu A \) for \( 1 \Phi_0 \),

\[ i_{max} = 26 \mu A (1/2 + A_{loop} / 2\pi) \]  
(A.19)
Appendix B

Lead-lag Filter

B.1 Introduction

The RC shunt filter, or lead-lag filter, provides additional FLL gain attenuation at high frequencies without affecting the readout bandwidth of the FDM system or incurring phase shifts at high frequencies. Figure B.1 shows a schematic of the lead-lag filter. It consists of three components, \( R \), the output impedance of a previous stage, and \( R_s \) and \( C_s \), the filter components. Although the lead-lag filter could be placed anywhere in the FLL, for convenience it is placed immediately at the output of the SQUID. The output impedance of the NIST 8-turn SSAs, \( \sim 100 \, \Omega \), then forms the \( R \) of the lead-lag filter. We then adjust \( R_s \) and \( C_s \).

The lead-lag filter is named for the way its impedance changes with frequency. At low frequencies, the output impedance \( R \) dominates and there is no reduction in gain. At intermediate frequencies, the lead-lag filter becomes an RC filter: the capacitive reactance shunts the output resistance \( R \). A phase “lag” is produced and the amplitude of the \( V_{out} \) begins to shrink. At high frequencies, the lead-lag filter becomes a purely resistive voltage-divider: the phase “lag” disappears and the full filter attenuation is achieved.

The lead-lag filter has two characteristic frequencies:

\[
 f_{\text{low}} = \frac{1}{(R + R_s)C_s} \tag{B.1} 
\]

is the frequency at which the FLL loop gain is reduced by 3 dB and

\[
 f_{\text{high}} = \frac{1}{R_sC_s} \tag{B.2} 
\]

is the frequency above which the full filter attenuation is achieved. The attenuation only depends on the resistive components of the lead-lag filter:

\[
 \frac{V_{out}}{V_{in}} = \frac{R_s}{R + R_s} = \frac{\gamma}{\gamma + 1} \tag{B.3} 
\]

where \( \gamma = R_s/R \) is the ratio of the resistive components.
To choose the filter parameters, we decide how much attenuation is needed, which sets $R_s$ (we assume that the lead-lag filter is immediately at the output of the SQUID which has $R \sim 100 \, \Omega$). Once the attenuation has been chosen, we choose $C_s$ to move $f_{\text{low}}$ above the FDM system bandwidth.

In practice we have found an attenuation of $1/6$ to $1/7$ is sufficient to provide stable performance with a set of eight LC filters attached to the input of the SQUID. We use $R_s = 17 \, \Omega$ and $C_s = 1.0 \, \text{nF}$, producing an attenuation of $1/7$, $f_{\text{low}} = 1.6 \, \text{MHz}$, and $f_{\text{high}} = 9.4 \, \text{MHz}$.

### B.2 Trade-Off: Attenuation vs. Phase Shift

Because the lead-lag filter is reactive over some bandwidth, it produces a phase shift in the feedback signal. The more attenuation we require, the larger the phase shift becomes. However, since the filter is purely resistive at both low and high frequencies, the phase shift is only incurred in a limited bandwidth. The frequency at which the phase shift is maximum is given by the relation:

$$f_{\text{max}} = \frac{1}{2\pi C_s \sqrt{R_s (R_s + R)}}$$

![Figure B.1. Schematic of lead-lag filter.](image-url)
and the maximum phase shift is given by:

$$\theta_{\text{max}} = \arctan \left( \frac{1}{2\sqrt{\gamma(\gamma + 1)}} \right)$$ (B.5)

The maximum phase shift only depends on the resistive components of the filter, not $C_s$. For the filter parameters listed in the previous section, a maximum phase shift of $48^\circ$ occurs at a frequency of 3.5 MHz. At this frequency, we have an additional phase shift of $\sim 90^\circ$ from the first stage amplifier roll-off, leaving $\sim 42^\circ$ of phase margin. More aggressively filtering (i.e. reducing $\gamma$) is not practical as it will reduce the phase margin unacceptably at 3.5 MHz.