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Possibility of detecting heavy neutral fermions in the Galaxy

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It is shown that heavy neutral fermions in the galactic halo could produce numerous detectable low-energy events in a "thermal" neutrino detector if the heavy-fermion—nucleon vector coupling is comparable in strength to the vector weak interaction. The conditions under which a detectable event rate could arise for fermions with purely axial-vector couplings are also discussed. In a silicon detector heavy-fermion events would be concentrated at low energies, and could be distinguished from solar or supernova neutrino events, which are expected to have a flatter energy spectrum. A measurement of the energy spectrum of heavy-fermion events could lead to a determination of the fermion mass, subject to astrophysical uncertainties concerning the velocity distribution of halo particles.

Recently, a new type of neutrino detector, which relies on the idea that even small neutrino energy losses ($\Delta E_\nu \gtrsim 1$ keV) in cold material ($T \sim 1-10$ mK) with a small specific heat could produce measurable temperature changes, has been proposed.[1] In the proposed detection scheme, the recoil energy from neutrino-electron or neutrino-nucleus scattering, which is rapidly thermalized, is responsible for heating the detector. Cabrera, Krauss, and Wilczek[1] have estimated that solar pp or Be^7 neutrinos could produce measurable events at a rate ~ 1/ton day in a Si detector, and that a burst of supernova neutrinos would produce ~ 10 simultaneous events in a 10-ton detector.

The purpose of this paper is to examine the possibility that such a detector can be used to observe heavy neutral fermions ($m \gtrsim 1$ GeV) in the Galaxy. Such particles, it has been suggested, could be a substantial component of the cosmological missing mass,[2,3] and would be expected to condense gravitationally, in particular, into galactic halos.[4] We will assume here that the extended massive halo believed to surround the Galaxy[5] is dominated by heavy fermions. The detection rates estimated here will therefore be upper bounds, with strict equalities pertaining only to the astrophysically interesting but possibly optimistic scenario in which heavy fermions are a significant constituent of the galactic halo. Similar calculations have been carried out independently by Goodman and Witten,[6] and Drukier, Freese, and Spergel.[6]

Heavy halo fermions, with typical momenta

$$\begin{aligned} p_{\rm rms} &= m v_h \\ &= 1\ \text{MeV}\, m(\text{GeV}) \left[\frac{v_h}{10^{-3}c}\right] \\ &= (2\times 10^{-11}\ \text{cm})^{-1} m(\text{GeV}) \left[\frac{v_h}{10^{-3}c}\right] \end{aligned} \tag{1}$$

for a halo velocity dispersion v_h, may interact coherently with detector nuclei of mass number A provided

$$m \lesssim 130\, A^{-1/3} \left[\frac{v_h}{10^{-3}c}\right]^{-1} \text{GeV}\,. \tag{2}$$

Assuming Eq. (2) to be satisfied, as must be the case if heavy fermions are to contribute significantly to the cosmological mass density,[2,3] coherent scattering with detector nuclei will be the dominant energy-loss mechanism.

We shall adopt a largely phenomenological description of the heavy-fermion—nucleus interaction. We write the heavy-fermion—nucleon interaction Lagrangian (in the point-interaction limit) as ($N = p$ for protons, n for neutrons)

$$\mathscr{L}_{f-N} = \sqrt{2}\, G_f J_f^\mu J_{N\mu}\,, \tag{3}$$

where the heavy-fermion current is

$$J_f^\mu = \bar\psi_{f,L}\gamma^\mu \psi_{f,L}\,, \tag{4}$$

for a left-handed Dirac or Majorana heavy fermion field $\psi_{f,L}$. Ignoring second-class currents we get,[7–9] for small momentum transfers,

$$\langle N' | J_N^\mu | N \rangle = \langle N' | J_N^\mu | N \rangle_V + \langle N' | J_N^\mu | N \rangle_A \tag{5}$$

for the matrix element of the nucleon current between states $|N\rangle$ and $|N'\rangle$, where in the nonrelativistic limit the vector (V) and axial-vector (A) components reduce to

$$\langle N' | J_N^0 | N \rangle_V = \frac{-i}{(2\pi)^3} u_{N'}^\dagger [g_V^{(N)}(0) + f_V^{(N)}(0)] u_N \tag{6}$$

and

$$\langle N' | \mathbf{J}_N | N \rangle_A = \frac{1}{(2\pi)^3} u_{N'}^\dagger [g_A^{(N)}(0) \boldsymbol{\sigma}] u_N \tag{7}$$

to leading order in $v/c \ll 1$. Equations (5)–(7) are of course identical to what is found in weak-interaction theory,[8,9] with the exception that here we regard the zero-momentum-transfer form factors $g_V^{(N)}(0)$, $f_V^{(N)}(0)$, and $g_A^{(N)}(0)$ as phenomenological parameters with unknown values (but presumably ~ 1 in general).

For simplicity, let us consider the interaction of a heavy fermion with an atomic nucleus, such as $_{14}\mathrm{Si}^{28}$, with total angular momentum $J=0$. For this case the axial-vector interaction, which transforms as a vector under rotations, makes no contribution to the nuclear matrix element. For coherent scattering it is then straightforward to show that[10]

$$d\sigma_V^{(D)} = \frac{G_f^2}{4\pi} [\tilde{g}_V^{(p)} Z + \tilde{g}_V^{(n)} (A-Z)]^2 m_R^2 \frac{dT}{T_{\max}} \tag{8a}$$

for Dirac fermions, and

$$d\sigma_V^{(M)} = \frac{G_f^2}{\pi} [\tilde{g}_V^{(p)} Z + \tilde{g}_V^{(n)} (A-Z)]^2 \times m_R^2 v^2 \left[1 - \frac{T}{T_{\max}}\right] \frac{dT}{T_{\max}} \tag{8b}$$

for Majorana fermions, where T is the nuclear recoil energy, whose maximum value is

$$T_{\max} = \frac{4mM_A}{(m+M_A)^2} E_f = \frac{2m^2 M_A}{(m+M_A)^2} v^2 \tag{9}$$

for an incident fermion kinetic energy $E_f = \frac{1}{2} m v^2$ and nuclear mass M_A. In Eq. (8), Z is the nuclear charge and A the total nucleon number,

$$\tilde{g}_V^{(N)} \equiv g_V^{(N)}(0) + f_V^{(N)}(0) \quad (N = n, p) \tag{10}$$

and

$$m_R = \frac{mM_A}{m+M_A} \tag{11}$$

is the reduced mass. Integrating over T gives the total scattering cross sections

$$\sigma_V^{(D)} = \frac{G_f^2}{4\pi} [\tilde{g}_V^{(p)} Z + \tilde{g}_V^{(n)} (A-Z)]^2 m_R^2 = 4.4 \times 10^{-39} [\tilde{g}_V^{(p)} Z + \tilde{g}_V^{(n)} (A-Z)]^2 \left[\frac{G_f}{G_F}\right]^2 \left[\frac{m_R}{1\ \mathrm{GeV}}\right]^2 \mathrm{cm}^2 \tag{12a}$$

and

$$\sigma_V^{(M)} = \frac{G_f^2}{2\pi} [\tilde{g}_V^{(p)} Z + \tilde{g}_V^{(n)} (A-Z)]^2 m_R^2 v^2 = 8.7 \times 10^{-45} [\tilde{g}_V^{(p)} Z + \tilde{g}_V^{(n)} (A-Z)]^2 \left[\frac{G_f}{G_F}\right]^2 \left[\frac{m_R}{1\ \mathrm{GeV}}\right]^2 \left[\frac{v}{10^{-3} c}\right]^2 \mathrm{cm}^2 , \tag{12b}$$

where $G_F \approx (290\ \mathrm{GeV})^{-2}$ is the Fermi weak coupling constant. For nominal values of the various parameters in Eq. (12), and $\tilde{g}_V^{(p)} \sim \tilde{g}_V^{(n)} \sim 1$, $\sigma_V^{(D)}$ is a good deal larger than, while $\sigma_V^{(M)}$ is comparable to, either neutrino-electron or neutrino-nucleus scattering cross sections for low-energy neutrinos ($E_\nu \sim 1$ MeV).

If $\tilde{g}_V^{(N)} = 0$ then the heavy-fermion—nucleon interaction is purely axial vector, and the cross section for (coherent) scattering of heavy fermions by a $J=0$ nucleus vanishes identically. This will be the case, for example, for photino-nucleus scattering, if the left- and right-handed scalar partners of the u and d quarks are equally massive.[11] For $\tilde{g}_V^{(N)} = 0$ but $g_A^{(N)} \neq 0$ we find, for scattering by a nucleus with $J \neq 0$,

$$d\sigma_A^{(D)} = \frac{G_f^2}{4\pi} \frac{|\mathcal{I}_A|^2}{2J+1} m_R^2 \frac{dT}{T_{\max}} \tag{13a}$$

for Dirac fermions, and

$$\frac{d\sigma_A^{(M)}}{dT} = 4 \frac{d\sigma_A^{(D)}}{dT} \tag{13b}$$

for Majorana fermions[12] where, using the Wigner-Eckart theorem[13]

$$\frac{\mathcal{I}_A}{\sqrt{2J+1}} = \left[\frac{J+1}{J}\right]^{1/2} \left[\left\langle J,J \left| g_A^{(p)}(0) \sum_p \sigma_z^{(p)} \right| J,J \right\rangle + \left\langle J,J \left| g_A^{(N)}(0) \sum_n \sigma_z^{(n)} \right| J,J \right\rangle \right] . \tag{14}$$

For nuclei with a single additional nucleon outside otherwise filled levels, $J=l\pm\frac{1}{2}$ and

$$\frac{\mathcal{J}_A}{\sqrt{2J+1}}=g_A^{(N)}(0)\left[\frac{J+1}{J}\right]^{1/2} \quad (J=l+\tfrac{1}{2})$$
$$=-g_A^{(N)}(0)\left[\frac{J}{J+1}\right]^{1/2} \quad (J=l-\tfrac{1}{2}) \tag{15}$$

for $N=n$ or p, while for nuclei with a single missing nucleon inside otherwise filled levels we get the same results multiplied by an overall factor -1 (Ref. 14). Integrating Eq. (13) over T gives the total cross sections

$$\sigma_A^{(D)}=\frac{G_f^2}{4\pi}\frac{|\mathcal{J}_A|^2}{2J+1}m_R^2$$
$$=4.4\times10^{-39}\frac{|\mathcal{J}_A|^2}{2J+1}\left[\frac{G_f}{G_F}\right]^2\left[\frac{m_R}{1\text{ GeV}}\right]^2\text{ cm}^2 \tag{16a}$$

and

$$\sigma_A^{(M)}=4\sigma_A^{(D)} \tag{16b}$$

which are much larger than low-energy neutrino-electron or neutrino-nucleus scattering cross sections unless G_f/G_F is appreciably smaller than unity.

In order to obtain estimated event rates for galactic fermions, we shall adopt a spherically symmetric isothermal model for the galactic halo.[15] The halo will be assumed to be nonrotating, so that the Solar System (and hence fermion detector) moves through the halo with a peculiar speed v_r, the rotation speed of the Sun about the galactic center. The event rate per detector nucleus for recoil energies between T and $T+dT$ is then $(dR/dT)dT$, where

$$\frac{dR}{dT}=\frac{(\rho_0/m)v_h}{\sqrt{2\pi}}\left[\frac{v_h}{v_r}\right]$$
$$\times\int_{v_{\min}(T)/v_h}^{\infty}du\,u^2\frac{d\sigma}{dT}(e^{-u_-^2/2}-e^{-u_+^2/2}) \tag{17}$$

with $d\sigma/dT$ a function of both recoil energy T and $u\equiv v/v_h$ in general [cf. Eqs. (8) and (13)], and $u_\pm\equiv u\pm v_r/v_h$. In Eq. (17) ρ_0 is the local mass density of the halo and

$$v_{\min}{}^2(T)=\frac{T(m+M_A)^2}{2m^2M_A}\ . \tag{18}$$

The halo velocity dispersion is

$$v_h{}^2=\frac{GM_0/R_0}{(-d\ln\rho/d\ln r)_{r=R_0}}\ , \tag{19}$$

where M_0 is the total halo mass within $r=R_0$, the solar (and hence detector) halo radius, and $-d\ln\rho/d\ln r=2$ for the singular isothermal sphere.[15] Recent determinations[16] are consistent with $\rho_0\approx0.01M_\odot\ \text{pc}^{-3}$ and $M_0\approx6\times10^{10}M_\odot$ at $R_0\approx8$ kpc, with

$$(-d\ln\rho/d\ln r)_{r=R_0}\approx2\ ,$$

implying[17] $v_h\approx130\ \text{km sec}^{-1}\approx0.43\times10^{-3}c$. The rotation speed will be taken to be[16] $v_r\approx250\ \text{km sec}^{-1}$. Event rates will only be estimated here for these fiducial values of v_r and v_h. Other contributions to Earth's peculiar motion through the halo (for example, the Solar System's oscillatory movement perpendicular to the galactic disk and the orbital motion of Earth around the Sun) will not be taken into account in this paper.

Substituting Eq. (8) in Eq. (17) gives

$$\frac{dR_V^{(D)}}{dT}=\frac{G_f{}^2\rho_0(M_A/m)[\tilde{g}_V^{(p)}Z+\tilde{g}_V^{(n)}(A-Z)]^2}{2(2\pi)^{3/2}v_h}$$
$$\times F_V^{(D)}\left[\frac{v_{\min}(T)}{\sqrt{2}v_h},\frac{v_r}{\sqrt{2}v_h}\right], \tag{20a}$$

for Dirac fermions where

$$F_V^{(D)}(x,y)=\frac{\sqrt{\pi}}{4y}[\text{erf}(x+y)-\text{erf}(x-y)] \tag{20b}$$

and

$$\frac{dR_V^{(M)}}{dT}=\frac{4G_f{}^2\rho_0(M_A/m)v_h[\tilde{g}_V^{(p)}Z+\tilde{g}_V^{(n)}(A-Z)]^2}{(2\pi)^{3/2}}$$
$$\times F_V^{(M)}\left[\frac{v_{\min}(T)}{\sqrt{2}v_h},\frac{v_r}{\sqrt{2}v_h}\right], \tag{20c}$$

for Majorana fermions where

$$F_V^{(M)}(x,y)$$
$$=\frac{1}{2y}\left[\frac{\sqrt{\pi}}{2}(\tfrac{1}{2}+y^2-x^2)[\text{erf}(x+y)-\text{erf}(x-y)]\right.$$
$$\left.+\left[\frac{x+y}{2}\right]e^{-(x-y)^2}-\left[\frac{x-y}{2}\right]e^{-(x+y)^2}\right], \tag{20d}$$

with erf(z) the usual error function. For Dirac fermions integrating Eq. (20a) over T gives a total event rate Γ_V per unit detector mass $M_D(\Gamma_V/M_D=R_V/M_A)$:

$$\frac{\Gamma_V^{(D)}}{M_D}=\frac{2G_f{}^2\rho_0v_h[\tilde{g}_V^{(p)}Z+\tilde{g}_V^{(n)}(A-Z)]^2}{(2\pi)^{3/2}}\frac{m/M_A}{(1+m/M_A)^2}G_V^{(D)}\left[\frac{v_r}{\sqrt{2}v_h}\right]$$
$$=2.5\times10^3\ \text{ton}^{-1}\ \text{day}^{-1}\left[\frac{G_f}{G_F}\right]^2[\tilde{g}_V^{(p)}Z+\tilde{g}_V^{(n)}(A-Z)]^2\frac{m/M_A}{(1+m/M_A)^2}\ , \tag{21a}$$

where

$$G_V^{(D)}(y)=\frac{1}{2y}[\sqrt{\pi}(\tfrac{1}{2}+y^2)\mathrm{erf}(y)+y\,e^{-y^2}]\approx 1.53 \tag{21b}$$

for $y=v_r/\sqrt{2}\,v_h\approx 1.36$. The corresponding event rate for Majorana fermions is

$$\Gamma_V^{(M)}=\frac{16G_f{}^2\rho_0 v_h{}^3[\tilde{g}_V^{(p)}Z+\tilde{g}_V^{(n)}(A-Z)]^2}{(2\pi)^{3/2}}G_V^{(M)}\left[\frac{v_r}{\sqrt{2}\,v_h}\right]$$
$$=8.0\times 10^{-3}\ \mathrm{ton}^{-1}\ \mathrm{day}^{-1}\left[\frac{G_f}{G_F}\right]^2[\tilde{g}_V^{(p)}Z+\tilde{g}_V^{(n)}(A-Z)]^2\frac{m/M_A}{(1+m/M_A)^2}\ , \tag{21c}$$

where

$$G_V^{(M)}(y)=\frac{1}{4y}[\sqrt{\pi}(y^4+3y^2+\tfrac{3}{4})\mathrm{erf}(y)+y(y^2+\tfrac{5}{2})e^{-y^2}]\approx 3.16\ . \tag{21d}$$

The average energy deposited per Dirac fermion event is

$$\langle T\rangle_V^{(D)}=\frac{4m^2M_A v_h{}^2}{(m+M_A)^2}H_V^{(D)}\left[\frac{v_r}{\sqrt{2}\,v_h}\right]=1.6\ \mathrm{keV}\left[\frac{m}{1\ \mathrm{GeV}}\right]\frac{m/M_A}{(1+m/M_A)^2}\ , \tag{22a}$$

where

$$H_V^{(D)}(y)=\frac{(y^4+3y^2+\tfrac{3}{4})\mathrm{erf}(y)+(y/\sqrt{\pi})(y^2+\tfrac{5}{2})e^{-y^2}}{2[(y^2+\tfrac{1}{2})\mathrm{erf}(y)+(y/\sqrt{\pi})e^{-y^2}]}\approx 2.07\ , \tag{22b}$$

while the average energy deposited per Majorana fermion event is

$$\langle T\rangle_V^{(M)}=\frac{4m^2M_A v_h{}^2}{(m+M_A)^2}H_V^{(M)}\left[\frac{v_r}{\sqrt{2}\,v_h}\right]=1.4\ \mathrm{keV}\left[\frac{m}{1\ \mathrm{GeV}}\right]\frac{m/M_A}{(1+m/M_A)^2}\ , \tag{22c}$$

where

$$H_V^{(M)}(y)=\frac{\sqrt{\pi}(y^6+15y^4/2+45y^2/4+\tfrac{15}{8})\mathrm{erf}(y)+(y^5+7y^3+33y/4)e^{-y^2}}{3[\sqrt{\pi}(y^4+3y^2+\tfrac{3}{4})\mathrm{erf}(y)+(y^3+5y/2)e^{-y^2}]}\approx 1.88\ . \tag{22d}$$

Integrated event rates at recoil energies $T\geq T_0$ are given in the Appendix. For Dirac fermions Eq. (A2) implies about 5×10^2 events with $T\geq 1$ keV per day in 1 ton of silicon if $G_f=G_F$, $\tilde{g}_V^{(p)}=\tilde{g}_V^{(n)}=1$, and $m=2$ GeV, and about 1×10^5 events per day if instead $m=5$ GeV, while Eq. (21a) implies total event rates, integrated over all recoil energies, equal to roughly 10^5 and 3×10^5 $\mathrm{ton}^{-1}\mathrm{day}^{-1}$, respectively. For Majorana fermions, Eq. (A3) implies about 4×10^{-3} events with $T\geq 1$ keV per day in 1 ton of silicon if $G_f=G_F$, $\tilde{g}_V^{(p)}=\tilde{g}_V^{(n)}=1$, and $m=2$ GeV, and about 4×10^{-1} events per day if instead $m=5$ GeV, while Eq. (21c) implies total event rates of about 4×10^{-1} $\mathrm{ton}^{-1}\mathrm{day}^{-1}$ and 8×10^{-1} ton day^{-1}, respectively. The event rate at $T\geq 1$ keV in a Si^{28} detector reaches its largest value, at[18] $m\approx 30$ GeV. If $G_f=G_F$ and $\tilde{g}_V^{(p)}=\tilde{g}_V^{(n)}=1$, the peak event rate is roughly 5×10^5 $\mathrm{ton}^{-1}\mathrm{day}^{-1}$ for Dirac fermions and 1.5 $\mathrm{ton}^{-1}\mathrm{day}^{-1}$ for Majorana fermions. (Since the average energy deposited is roughly 10 keV for Majorana or Dirac fermions with $m\approx 30$ GeV, the total event rate is essentially the same as the event rate at $T\geq 1$ keV.) Smaller event rates would of course be recorded if $G_f<G_F$. For example, taking $\tilde{g}_V^{(p)}=\tilde{g}_V^{(n)}=1$, Eq. (A2) implies $\lesssim 1$ event per day at $T\geq 1$ keV for Dirac fermions with $m\gtrsim 5$ GeV if $G_f\lesssim 10^{-3}G_F$, corresponding to $M_B\gtrsim 3000$ GeV for $G_f\equiv(e/M_B)^2$.

For heavy fermions whose coupling to nucleons are purely axial vector, the event rate in a pure Si^{28} detector would vanish identically. However, naturally occurring silicon consists of 4.7% Si^{29}, which has one neutron outside otherwise filled shells, implying a nonzero scattering cross section.[19] Substituting Eq. (13) into Eq. (17) gives, for Dirac fermions,

$$\frac{dR_A^{(D)}}{dT}=\frac{G_f{}^2\rho_0(M_A/m)}{2(2\pi)^{3/2}v_h}\frac{|\mathcal{J}_A|^2}{2J+1}F_A\left[\frac{v_{\min}(T)}{\sqrt{2}\,v_h},\frac{v_r}{\sqrt{2}\,v_h}\right], \tag{23a}$$

and

$$\frac{dR_A^{(M)}}{dT}=4\frac{dR_A^{(D)}}{dT}\ , \tag{23b}$$

where

$$F_A(x,y)=F_V^{(D)}(x,y)\ . \tag{23c}$$

Integrating Eq. (23) over T gives a total event rate per unit detector mass

$$\frac{\Gamma_A^{(D)}}{M_D}=\frac{2G_f{}^2\rho_0 v_h}{(2\pi)^{3/2}}\frac{f_J|\mathcal{J}_A|^2}{2J+1}\frac{m/M_A}{(1+m/M_A)^2}G_A\left[\frac{v_r}{\sqrt{2}v_h}\right]=2.5\times10^2\ \text{ton}^{-1}\text{day}^{-1}\left[\frac{G_f}{G_F}\right]^2\left[\frac{f_J}{0.1}\right]\frac{|\mathcal{J}_A|^2}{2J+1}\frac{m/M_A}{(1+m/M_A)^2}, \tag{24a}$$

for Dirac fermions, and $\Gamma_A^{(M)}/M_D=4\Gamma_A^{(D)}/M_D$ for Majorana fermions, where

$$G_A(y)=G_V^{(D)}(y)\approx1.53\ , \tag{24b}$$

and f_J is the fraction of the detector mass in $J\neq0$ nuclei. The average energy deposited per event is

$$\langle T\rangle_A=\frac{19m^2M_Av_h{}^2}{3(m+M_A)^2}H_A\left[\frac{v_r}{\sqrt{2}v_h}\right]=1.6\ \text{keV}\left[\frac{m}{1\ \text{GeV}}\right]\frac{m/M_A}{(1+m/M_A)^2}\ , \tag{25a}$$

for both Majorana and Dirac fermions where

$$H_A(y)=H_V^{(D)}(y)\approx2.07\ . \tag{25b}$$

Integrated event rates at recoil energies $T\geq T_0$ may be found using the Appendix. For Dirac fermions Eq. (A2) implies[20] about 2×10^{-2} events with $T\geq1$ keV per day in 1 ton of silicon if $g_A^{(n)}(0)=1$, $G_f=G_F$, and $m=2$ GeV, and about 2×10^1 events per day if instead $m=5$ GeV while Eq. (24) implies total event rates, integrated over all recoil energies, equal to roughly 2×10^1 and 5×10^1 $\text{ton}^{-1}\text{day}^{-1}$, respectively. The event rate at $T\geq1$ keV reaches its largest value at $m\approx30$ GeV. For Dirac fermions, the peak event rate is roughly 9×10^1 $\text{ton}^{-1}\text{day}^{-1}$ if $g_A^{(n)}(0)=1$, $G_f=G_F$, and $f_J=0.047$. (As before, the total event rates and event rates at $T\geq1$ keV are essentially the same for $m\approx30$ GeV.) Event rates for Majorana fermions are, from Eq. (23b), a factor of 4 larger. All of the event rates estimated above could be considerably larger for $G_f\gg G_F$, and vice versa.

While the main source of uncertainty in the rates estimated above is the largely unconstrained microphysics of the heavy fermion interactions with nucleons, it is important to bear in mind some astrophysical ambiguities as well. The main difficulty arises from the essentially unknown velocity distribution of the halo particles, which we have taken to be a continuous Maxwell-Boltzmann distribution with a single velocity dispersion v_h. Even if our implicit assumption of a locally isotropic velocity distribution is correct, one would expect a sharp cutoff for speeds greater than the local escape speed $v_e(r_0)$. Using a truncated rather than continuous Maxwell-Boltzmann distribution would result in generally lower event rates. However, as we shall argue below, we do not expect that including a velocity cutoff will substantially alter our results, at least for moderately large fermion masses.

A precise value of $v_e(r_0)$ is not known because it depends critically on the extent of the flat portion of the galactic rotation curve.[21] [Indeed, for a truly unbounded isothermal sphere, $v_e(r)$ is infinite at all r, and even for a tidally limited isothermal profile, $v_e(r)/v_h$ may be very large.] If the galactic rotation curve is flat out to $r\gtrsim2r_0$, which is not implausible, then[21] $v_e(r_0)/v_r\gtrsim[2(\ln2+1)]^{1/2}$ or[22] $v_e(r_0)\gtrsim460\ \text{km sec}^{-1}$. At low values of m, where $v_{\min}(T_0)/v_h\sim$ a few, heavy fermion events at $T\geq T_0$ are mainly due to particles with halo speeds near $v_{\min}(T_0)-v_r$, the lowest possible value. As long as $[v_{\min}(T_0)-v_r]^2$ is significantly smaller than $v_e{}^2(r_0)$ our estimated event rates should be reasonably accurate. For $m=2$ GeV, $[v_{\min}(1\ \text{keV})-v_r]^2\approx6.6v_h{}^2$, as opposed to $v_e{}^2(r_0)\gtrsim12.5v_h{}^2$, so this condition is satisfied.[23] At larger values of m, the event rate above 1 keV is due to a larger range of halo speeds. Roughly speaking we expect our estimated event rates to be adequate provided $v_{\min}{}^2(T_0)\lesssim v_e{}^2(r_0)+v_r{}^2$, which is the same as requiring that the typical halo kinetic energy needed to produce an event not exceed the minimum kinetic energy for escape. Based on this criterion, our calculated event rates should already be reliable at $m=5$ GeV [$v_{\min}{}^2(1\ \text{keV})\approx3.9v_h{}^2$ as opposed to $v_e{}^2(r_0)+v_r{}^2\gtrsim16.2v_h{}^2$] and should be very accurate at larger values of m [e.g., $v_{\min}{}^2$ (1 keV)$\approx v_h{}^2$ for $m\approx10$ GeV]. All in all, the average event rates calculated above are probably reasonably accurate over the entire range of fermion masses considered ($m\gtrsim2$ GeV), and are certainly dependable estimates at $m\gtrsim5$ GeV.

Conservatively, then, we conclude that galactic heavy fermions would produce numerous detectable events in a one-ton thermal detector of the type described in Ref. 1 if the heavy neutral fermion couplings to nucleons are at least comparable in strength to the weak interaction and if the fermion mass $m\gtrsim2$–3 GeV. This would certainly be the case for sufficiently heavy neutral leptons, which may be either Majorana or Dirac fermions. For photinos, which are generally thought to be Majorana fermions,[3,11] vector couplings roughly as strong as the weak interaction could arise for left- and right-handed scalar-quark masses satisfying[24] $|1/m_L{}^2-1/m_R{}^2|\sim(100\ \text{GeV})^{-2}$, while far stronger axial-vector couplings, corresponding to $m_L\approx m_R\ll100$ GeV, cannot be ruled out.[3]

Heavy fermion events would be concentrated at low energies, and could be distinguished from low-energy neutrino events by their energy spectrum and their possibly large frequency of occurrence. The optimum observing strategy for simply detecting halo fermions would be to use a detector enriched in $J\neq0$ nuclei. Such a detector would be especially sensitive to Dirac heavy leptons, through the coherent vector interaction, but would also register substantial event rates for Majorana heavy leptons or photinos, for which the axial-vector coupling would dominate. Because the expected energy spectrum is essentially the same for Majorana and Dirac fermions in such a detector (ignoring the relatively small number of events due to the suppressed coherent vector coupling of Majorana fermions), one could not distinguish between these two possibilities for the fermion mass eigenstates from observations with a single $J\neq 0$ nuclear species. For this purpose one could compare the results of observations made with $J\neq 0$ nuclei of different nuclear charges Z and nucleon members A, since only Dirac fermions scatter coherently to a significant degree. Alternatively, one could use a detector enriched in $J\neq 0$ nuclei to distinguish between Majorana and Dirac fermions, as the expected energy spectra [for example, Eqs. (20a), (20b) and (20c), (20d) for an isothermal halo model] are reasonably different for the two possibilities.

Assuming that the isothermal distribution accurately describes the galactic halo locally,[25] one could in principle determine the fermion mass m from a fit of Eqs. (20) and/or (23) to the observed energy spectrum of heavy fermion events. Moreover, such an analysis would yield independent determinations of v_h and v_r (more precisely, Earth's peculiar velocity relative to the halo), which would be of great interest astrophysically. [For a truncated isothermal distribution, one could also obtain information on $v_e(r_0)$, although the dependence of the predicted event rates on this parameter is weak at $m \gtrsim 2$–3 GeV.] The significance of the resulting derived parameters would be determined by the energy resolution of the data and by the appropriateness of the isothermal halo model.

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APPENDIX: EVENT RATES AT $T \geq T_0$

The total event rate at recoil energies above a threshold value T_0 is

$$\frac{\Gamma(\geq T_0)}{M_D}=\frac{1}{M_A}\int_{T_0}^{\infty} dT\frac{dR}{dT}\ , \tag{A1}$$

where dR/dT is given by either Eqs. (20) or Eq. (23). Plugging Eqs. (20a) and (20b) into Eq. (A1) gives

$$\frac{\Gamma_V^{(D)}(\geq T_0)}{M_D}=\frac{\Gamma_{V,0}^{(D)}}{M_D}K_V^{(D)}\left[\frac{v_{\min}(T_0)}{\sqrt{2}\,v_h},\frac{v_r}{\sqrt{2}\,v_h}\right]\ , \tag{A2a}$$

where

$$\frac{\Gamma_{V,0}^{(D)}}{M_D}=\frac{2G_f^2\rho_0 v_h[\tilde{g}_V^{(p)}Z+\tilde{g}_V^{(n)}(A-Z)]^2}{(2\pi)^{3/2}}\frac{m/M_A}{(1+m/M_A)^2} \tag{A2b}$$

and

$$K_V(x,y)=\frac{1}{2y}\left[\frac{\sqrt{\pi}}{2}(y^2-x^2+\tfrac{1}{2})[\operatorname{erf}(x+y)-\operatorname{erf}(x-y)]+\left[\frac{x+y}{2}\right]e^{-(x-y)^2}-\left[\frac{x-y}{2}\right]e^{-(x+y)^2}\right]. \tag{A2c}$$

Taking $x=0$ in Eq. (A2) gives Eqs. (21a) and (21b). Plugging Eqs. (20c) and (20d) into Eq. (A1) gives

$$\frac{\Gamma_V^{(M)}(\geq T_0)}{M_D}=\frac{\Gamma_{V,0}^{(M)}}{M_D}K_V^{(M)}\left[\frac{v_{\min}(T_0)}{\sqrt{2}\,v_h},\frac{v_r}{\sqrt{2}\,v_h}\right]\ , \tag{A3a}$$

where

$$\frac{\Gamma_{V,0}^{(M)}}{M_D}=\frac{16G_f^2\rho_0 v_h^3}{(2\pi)^{3/2}}[\tilde{g}_V^{(p)}Z+\tilde{g}_V^{(n)}(A-Z)]^2\frac{m/M_A}{(1+m/M_A)^2} \tag{A3b}$$

and

$$K_V^{(M)}(x,y)=\frac{1}{8y}\{\sqrt{\pi}(y^4+3y^2+x^4-2x^2y^2-x^2+\tfrac{3}{4})[\operatorname{erf}(x+y)-\operatorname{erf}(x-y)]+[(x+y)(y^2-x^2+\tfrac{3}{2})+y]e^{-(x-y)^2}-[(x-y)(y^2-x^2+\tfrac{3}{2})-y]e^{-(x+y)^2}\}\ . \tag{A3c}$$

Taking $x=0$ in Eq. (A3) gives Eqs. (21c) and (21d). Plugging Eq. (23) into Eq. (A1) gives

$$\frac{\Gamma_A^{(D)}(\geq T_0)}{M_D}=\frac{\Gamma_{A,0}^{(D)}}{M_D}K_V^{(D)}\left[\frac{V_{\min}(T_0)}{\sqrt{2}\,v_h},\frac{v_r}{\sqrt{2}\,v_h}\right]\ , \tag{A4a}$$

where

$$\frac{\Gamma_{A,0}^{(D)}}{M_D}=\frac{2G_f^2\rho_0 v_h}{(2\pi)^{3/2}}\frac{f_J\,|\mathcal{F}_A|^2}{2J+1} \tag{A4b}$$

and $K_V^{(D)}(x,y)$ is given by Eq. (A2b). Taking $x=0$ in Eq. (A4) gives Eq. (24).

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[13]We are tacitly assuming that the nuclear shell model is an adequate approximation. The Wigner-Eckart theorem is discussed in general in K. Gottfried, *Quantum Mechanics* (Benjamin, New York, 1966), Sec. 36.3. Particular applications to the nuclear shell model are discussed in A. deShalit and H. Feshbach, *Theoretical Nuclear Physics,* Volume 1: Nuclear Structure (Wiley, New York, 1974), Appendix A.

[14]See deShalit and Feshbach (Ref. 13).

[15]See, for example, S. Chandrasekhar, *Principles of Stellar Dynamics* (Dover, New York, 1960), Sec. 5.8.

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[17]It is interesting to note that the velocity dispersions of various halo objects, although generally anisotropic, are in good agreement with the result of Eq. (19). See K. C. Freeman, in *The Milky Way Galaxy,* edited by H. van Woerden, R. J. Allen, and W. B. Burton (I.A.U. Symposium No. 106) (Reidel, Dordrecht, 1985). A recent study of the kinematics of 29 distant RR Lyrae stars in the halo is consistent with an isotropic velocity dispersion of 109 ± 16 km sec^{-1}. See A. Saha, Astrophys. J. **289**, 310 (1985). The best fit of a model with an isotropic velocity dispersion to the kinematics of 66 globular clusters gives $v_h \approx 118$ km sec^{-1} at $r=9$ kpc, but is not isothermal ($v_h \propto r^{0.2}$). See C. Frenk and S. D. M. White, Mon. Not. R. Astron. Soc. **193**, 295 (1980). These results indicate the range of uncertainty in galactic halo models, but support the value $v_h \approx 130$ km sec^{-1} adopted here, at least crudely.

[18]Such large values of m may, however, result in a very low present-day cosmological abundance of heavy fermions. See Refs. 2 and 3. Solar System constraints may also rule out Dirac neutrinos with $m \gtrsim 10$ GeV. See L. M. Krauss, M. Srednicki, and F. Wilczek, Report No. HUTP-85/A007, 1985 (unpublished); and K. Freese, CFA Report No. 2108, 1985 (unpublished).

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[20]Although Si^{29} is a deformed nucleus, and may only be accurately understood in terms of an axisymmetric nuclear potential (Nilsson Hamiltonian) rather than a spherically symmetric shell model potential, we use the matrix element for its $2S_{1/2}$ parent state, $|\mathcal{J}_A|^2/(2J+1)=(J+1)/J=3$, in estimating event rates. That this may be reasonably accurate is indicated by the fact that even relatively large distortions of the $2S_{1/2}$ parent shell-model state lead to little change in the orbital energy. See Enge (Ref. 18) Sec. 6.8 and deShalit and Feshbach (Ref. 13) Sec. VI.10, and references therein.

[21]The escape velocity at radius r is given by $v_e^2(r) = -2[\phi(r)-\phi(r_T)]$, where r_T is the tidal radius of the halo, beyond which halo particles can no longer be considered bound to the Galaxy. In terms of the rotation velocity $v_r(r)$,

$$v_e^2(r)=2\int_r^{r_T} dr' v_r^2(r')/r' \ .$$

Assuming a flat rotation curve out to $r=\lambda r_0$, and $r_T \gg \lambda r_0$ gives $v_e^2(r_0)/v_r^2 > 2(\ln\lambda+1)$, since $v_r^2(r) > v_r^2(\lambda r_0/r)$ at $\lambda r_0 < r < r_T$. See Ref. 15.

[22]Observations of high-velocity stars in the solar neighborhood imply $v_e(r_0) \gtrsim 400$–450 km sec^{-1}. See D. Lynden-Bell and D. N. C. Lin, Mon. Not. R. Astron. Soc. **181**, 37 (1977), and references therein.

[23]Perhaps more relevant is a comparison of the Boltzmann factors $f_{\min}=\exp\{-[v_{\min}(T_0)-v_r]^2/2v_h^2\}$ and $f_e=\exp[-v_e^2(r_0)/2v_h^2]$. For $m=2$ GeV and $T_0=1$ keV, $f_{\min}/f_e \gtrsim 20$, indicating that our calculated event rate at 2 GeV is accurate to much better than a factor of 2 for our adopted galactic parameters. However, $f_{\min}/f_e \lesssim 1$ for $m \lesssim 1.7$ GeV, indicating that the results at low m are sensitive to the values of v_h, v_r, and $v_e(r_0)$ used. For larger masses, $m \gtrsim 5$ GeV, the dependences on these parameters are expected to be relatively weak, for reasons discussed in the text.

[24]Parity violation in nuclear interactions may constrain $|1/m_L^2-1/m_R^2| \lesssim (100 \text{ GeV})^{-2}$ unless u and d scalar quarks are themselves degenerate in mass. See M. Suzuki, Phys. Lett. **115B**, 40 (1982), and also J. Ellis and D. V. Nanopoulos, *ibid.* **110B**, 44 (1982) for a discussion of mass degeneracy among various flavors of scalar quarks. For photino masses $m \gtrsim 5$ GeV, the event rate due to vector coupling is still appreciable for $G_f/G_F \gtrsim 10^{-2}$, corresponding very roughly to $|1/m_L^2-1/m_R^2| \gtrsim (1000 \text{ GeV})^{-2}$ for u and d scalar quarks.

[25]It is perhaps worth noting that the local velocity distribution on laboratory space and time scales need not be even approximately Maxwellian even if an isothermal model adequately describes the large-scale properties of the galactic halo as a whole. This is because violent relaxation of collisionless halo particles leaves their fine-grained phase-space density unchanged even though it may cause the coarse-grained distribution to approach a (truncated) Maxwellian. The separation between the length scales on which the coarse and fine-grained distributions apply is not well understood theoretically. If this critical length scale is large compared to $v_h t_{\text{expt}} \approx 10^{15}$ cm t_{expt}(yr) for an experiment lasting a time t_{expt}, then the incident heavy fermions could even be monoenerget-

ic, in which case the observed energy spectrum would be directly proportional to the differential cross sections, Eqs. (8a), (8b), (13a), and (13b). It is, of course, true that the event rates and energy spectra in an isothermal halo would, over sufficiently long time scales, approach the expressions derived in this paper.