Superfluid Dark Matter

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Introduction

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It faces some challenges at galactic scales.

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$$\rho_{\mathrm{NFW}} = \frac{\rho_s}{\frac{r}{r_s}\left(1 + \frac{r}{r_s}\right)^2}$$

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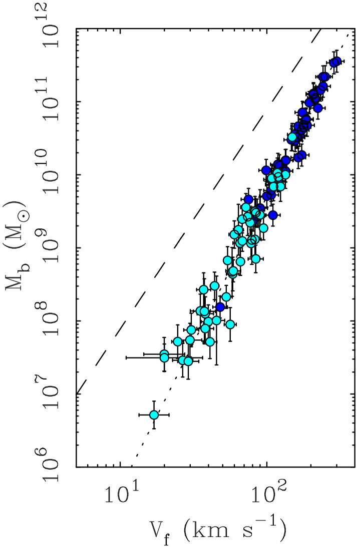
"Missing satellite"/"Too big to fail" problem

BTFR:



$M_{\rm b} \sim v_{\rm c}^4$

CDM: $M_{\rm vir} \sim v_{vir}^3$

Famaey and McGaugh '12

MOND:

Milgrom '83

$$a = \begin{cases} a_{\mathrm{N}} & a_{\mathrm{N}} \gg a_0 \\ \sqrt{a_{\mathrm{N}} a_0} & a_{\mathrm{N}} \ll a_0 \end{cases}$$

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MOND fails on cosmological scales.

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The theory must avoid co-existence of DM and MOND behavior.

It would be much more compelling if these two components somehow had a common origin.

Unified Framework:

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$$\mathcal{L}_{\text{MOND}} \sim \left[(\vec{\nabla}\varphi)^2\right]^{3/2}$$

vs

$$\mathcal{L}_{\text{superfluid}} = P(X) \qquad \text{with} \qquad X \equiv \mu + \dot{\varphi} - \frac{(\vec{\nabla}\varphi)^2}{2m}$$

P depends on the properties of the superfluid.

Fractional Powers?

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Unitary Fermi Gas is described by

$$\mathcal{L}_{\text{UFG}} \sim X^{5/2}$$

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Upon BEC, it exhibits the superfluidity.

EFT for phonons is

$$\mathcal{L} \sim X^{3/2}$$

BEC of Dark Matter

For degeneracy

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Applied at virialization for Milky Way

$$m < 2 \text{ eV}$$

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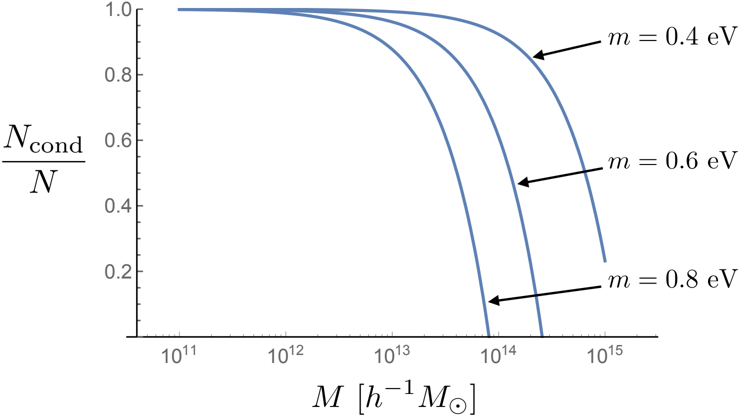
$$\boxed{m = 0.6\ \mathrm{eV} \qquad \frac{\sigma}{m} \gtrsim 0.1\ \frac{\mathrm{cm}^2}{\mathrm{g}}}$$

Halo as a Superfluid at Finite Temperature



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DM in sub-eV mass range must be produced out of equilibrium. For instance, through axion-like vacuum displacement mechanism.

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Superfluid Properties:

Low energy degrees of freedom are phonons.
In general

$$\mathcal{L} = P(X), \qquad \text{with} \qquad X \equiv \mu + \dot{\varphi} - \frac{(\vec{\nabla}\varphi)2}{2m}$$

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To mediate MOND force, phonons must couple to baryons

$$\mathcal{L}_{\rm int} = -\alpha \frac{\Lambda}{M_{\rm pl}} \varphi \rho_{\rm b}$$

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The pressure of the condensate is

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The equation of state is

$$P = \frac{\rho^3}{12\Lambda^2 m^6}$$

The sound speed is given by $c_s = \sqrt{\frac{2\mu}{m}}$

Halo Profile:

Assuming hydrostatic equilibrium

$$\rho(r) \simeq \rho_0 \cos\left(\frac{\pi}{2}\frac{r}{R}\right); \qquad r \leq R$$

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For fiducial values $m = 0.6$ eV and $\Lambda = 0.2$ meV, for $M_{\rm DM} = 10^{12} M_{\odot}$, we have

$$R \simeq 125 \text{ kpc}$$

The superfluid scenario provides the simple resolution to the cusp-core and "too-big-to-fail" problems.

Phonon-Mediated Force Between Baryons

For large phonon gradients

$$a_\phi(r) = \sqrt{\frac{\alpha^3\Lambda^2}{M_{\rm Pl}}\frac{G_{\rm N}M_{\rm b}(r)}{r^2}}$$

$$\frac{\alpha^3\Lambda^2}{M_{\rm Pl}} \equiv a_0 \sim H_0$$

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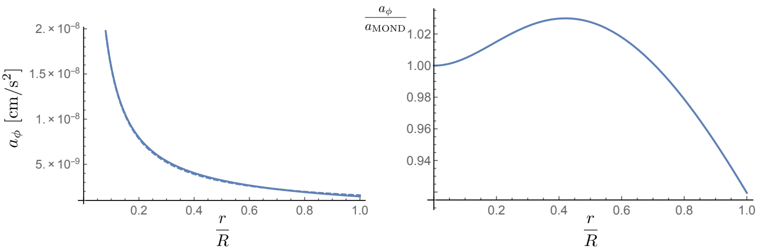

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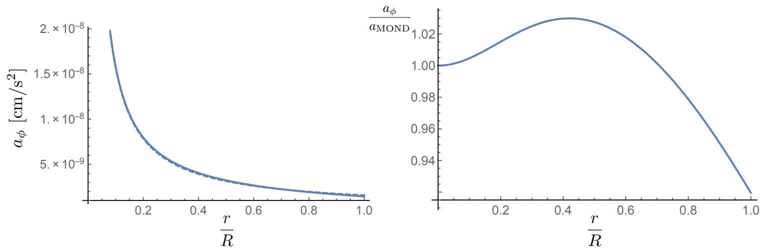

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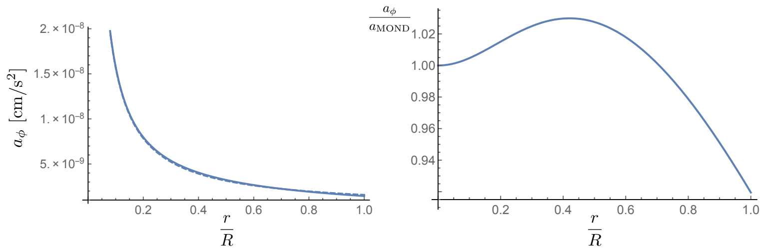

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For the stability of perturbations we need to invoke the finite temperature effects.

Validity of EFT and the Solar System

MOND regime corresponds to large phonon gradients

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High derivative terms need to be small. This is so as long as $r \gg \Lambda_s^- 1$.

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$$r \gg 0.2 \left(\frac{M_{\rm b}}{10^{11}\, M_\odot}\right)^{1/2} \left(\frac{m}{\rm eV}\right)^{-1/2} \left(\frac{\Lambda}{\rm meV}\right)^{-5/6} \;{\rm kpc}$$

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Applied to the Sun

$$r \gg 250 \left(\frac{m}{\mathrm{eV}}\right)^{-1/3} \left(\frac{\Lambda}{\mathrm{meV}}\right)^{-5/9} \;\mathrm{AU}\,,$$

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Recall that

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Gravitational Lensing

We might need to couple baryons to

$$\tilde{g}_{\mu\nu} \simeq g_{\mu\nu} - \frac{2\alpha}{M_{\mathrm{pl}}}\varphi\Big(\gamma g_{\mu\nu} + (1+\gamma)u_\mu u_\nu\Big)$$

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In our case we have DM, unlike TeVeS, and hence we may even get away with $\gamma = -1$.

Merging Clusters:

For BEC in galaxies we need

$$\frac{\sigma}{m} \gtrsim 0.1 \ \frac{\mathrm{cm}^2}{\mathrm{g}}$$

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The bound may not even apply because of the superfluidity.
e.g. for Bullet Cluster $c_s \sim v_{\mathrm{infall}}$.

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If the infall velocity is sub-sonic, the superfluid components should once again pass through each other with negligible friction, however the normal components should be slowed down.

Other Consequences:

- Vortices
- Galaxy Mergers
- Reduced Dynamical Friction
- Globular Clusters
- Tri-axial DM Halos

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Next Step: Finding a viable microscopic theory.