Cosmological Imprints of Dark Matter Produced During Inflation

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Isocurvature Sample

High-energy theory motivated:

Multi-field inflation: Axenides, Brandenberger, **Turner** 83; Linde 85; Starobinsky 85; Silk, **Turner** 87; Polarski, Starobinsky 94; Linde, Mukhanov 97; Langlois 99

Axions: Turner, Wilczek, Zee 83; Steinhardt, **Turner** 83; Axenides, Brandenberger, **Turner** 83; Linde 84, 85; Seckel, **Turner** 85; Efstathiou, Bond 86; **Hogan**, Rees 88; Lyth 90; Linde, Lyth 90; **Turner**, Wilczek 91; Linde 91; Lyth 92; **Kolb**, Tkachev 94; Fox, Pierce, Thomas 04; Beltran, Garcia-Bellido, Lesgourgues 06; Hertzberg, Tegmark, Wilczek 08; Kasuya, Kawasaki 09; Marsh, **Grin**, Hlozek, Ferreira 13; Choi, Jeong, Seo 14

B-genesis/Affleck-Dine: Bond, **Kolb**, Silk 82; Enqvist, McDonald 99, 00; Kawasaki, Takahashi 01; Kasuya, Kawasaki, Takahashi 08; Harigaya, Kamada, Kawasaki, Mukaida, Yamada 14; Hertzberg, Karouby 14

SUSY Moduli: Yamaguchi 01; Moroi and Takahashi 01; Lemoine, Martin, Yokoyama 09; Iliesiu, Marsh, Moodley, Watson 13

WIMPZILLAs: Chung, **Kolb**, Riotto, Senatore 04; Chung, **Yoo** 11; Chung, **Yoo**, Zhou 13

fermionic isocurvature perturbations: Chung, **Yoo**, Zhou 13

dark energy isocurvature: Malquarti, Liddle 02; Gordon, **Hu** 04

some recent observation focus (many before this time period):

21 cm: Gordon, Pritchard 09; Kawasaki, Sekiguchi, Takahashi 11; Takeuchi, Chongchitnan 13; Sekiguchi, Tashiro, Silk, Sugiyama 13

inhomogeneous baryons and BBN: Holder, Nollett, Engelen 09

compensated isocurvature perturbations: **Grin**, Dore, Kamionkowski 11

CMB: Hikage, Kawasaki, Sekiguchi, Takahashi 12; Planck 2013 results XXII; Chlub and **Grin** 13; **Grin**, Hanson, Holder, Dore, Kamionkowski 13; Kawasaki and Yokoyama 14

Fluid dynamics and general field theory parameterizations:

Bardeen 1980; review of Kodama, Sasaki 84; review of Malik, Wands 09; **Kolb** and **Turner**, EARLY UNIVERSE

+ Many more papers on curvaton and non-Gaussianities; apologies for omissions

Part I

What is the motivation for studying cosmological imprints of dark matter (DM) produced during inflation?

A **theoretically well-motivated** class of DM called **non-thermal** DM needs a probe of its production mechanism.

Dark matter (DM) explains a large number of cosmological data economically.

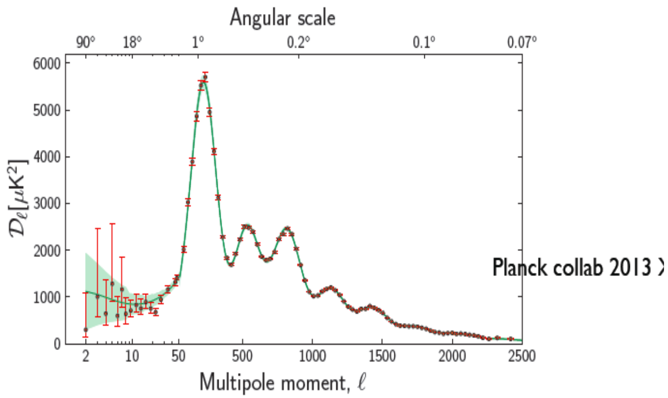
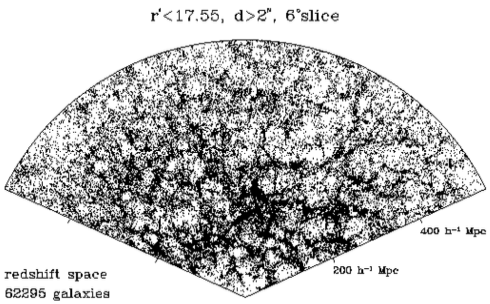
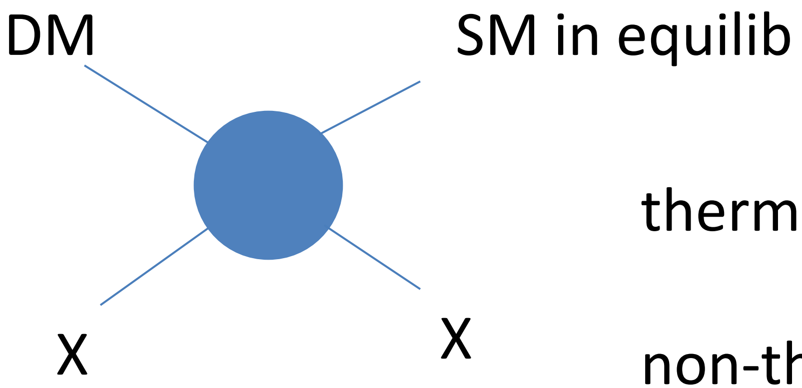


Figure 37. The 2013 *Planck* CMB temperature angular power spectrum. The error bars include cosmic variance, whose magnitude is indicated by the green shaded area around the best fit model. The low-ℓ values are plotted at 2, 3, 4, 5, 6, 7, 8, 9.5, 11.5, 13.5, 16, 19, 22.5, 27, 34.5, and 44.5.



From the point of view of early universe cosmology, there are two types of DM:

thermal and $\boxed{\text{non-thermal}}$.



thermal = DM # changing in equilib

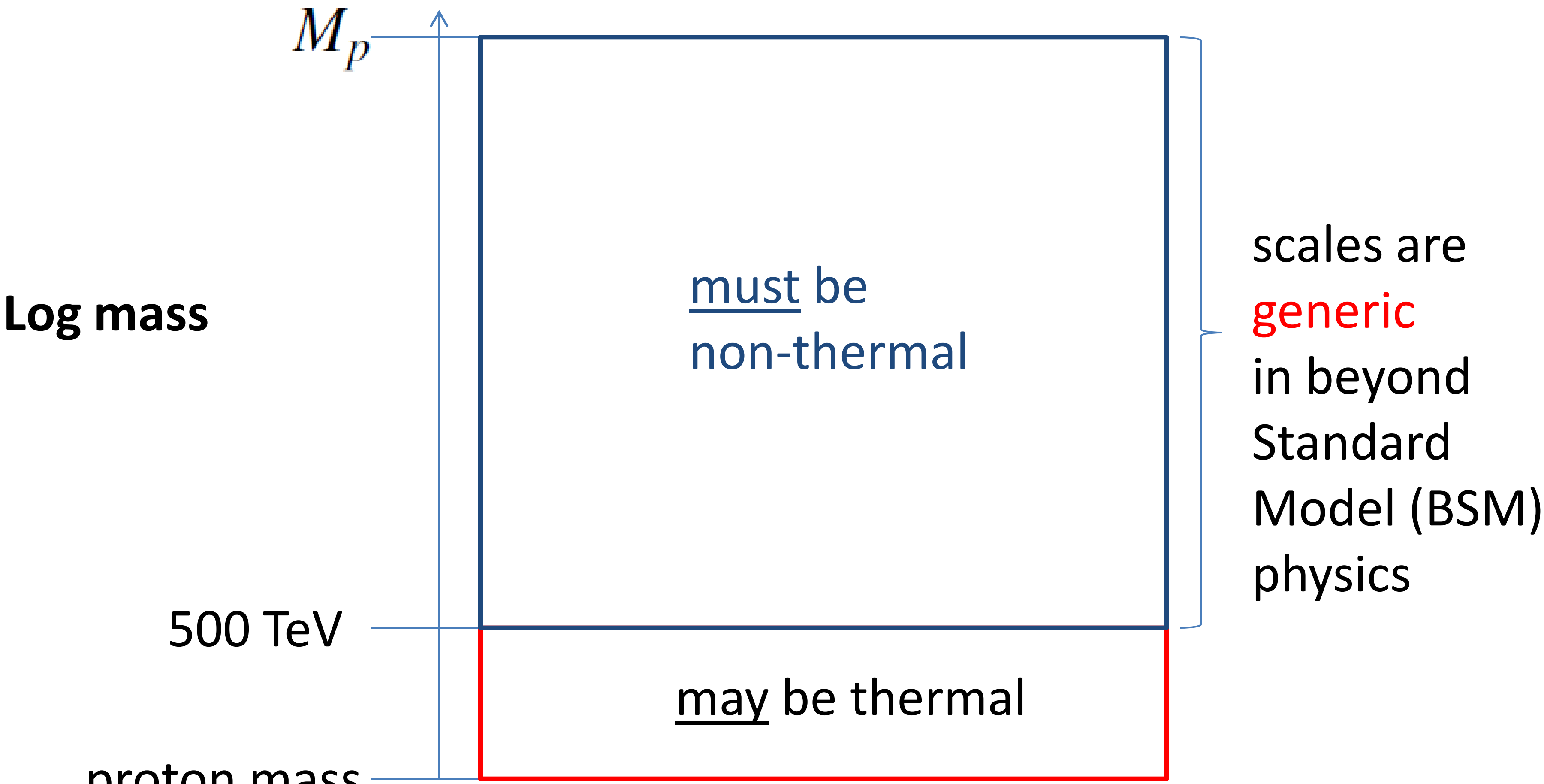
non-thermal = DM # changing out of equilib

In the hot big bang plasma, what is the number density of DM?

thermal	non-thermal
$n_{DM}(T,\mu)$	production mechanism **dependent**

Theoretical appeal of non-thermal DM?

1) Mass space is large for non-thermal dark matter



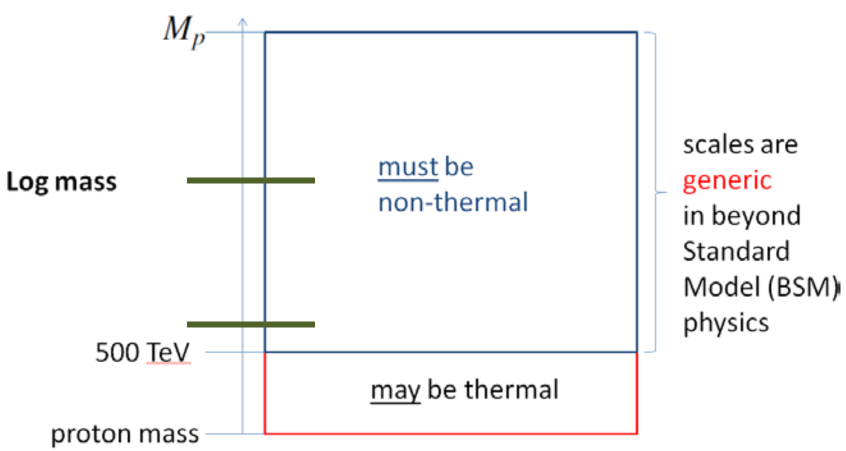
[Griest, Kamionkowski 89;
DC, Kolb, Riotto 98]

example:

Supersymmetry is one of the best motivated BSM paradigms.

It seems to **require** a **hidden sector** to embed the Standard Model of particle physics.

Hidden sector scales are in the non-thermal mass region



Furthermore, they are natually non-thermalizing with the SM fields:
i.e. hidden

Some Examples of BSM models

J. R. Ellis, J. L. Lopez, and D. V. Nanopoulos, *Confinement of fractional charges yields integer charged relics in string models*, *Phys.Lett.* **B247** (1990) 257.

K. Benakli, J. R. Ellis, and D. V. Nanopoulos, *Natural candidates for superheavy dark matter in string and M theory*, *Phys.Rev.* **D59** (1999) 047301, [hep-ph/9803333].

A. Kusenko and M. E. Shaposhnikov, *Supersymmetric Q balls as dark matter*, *Phys.Lett.* **B418** (1998) 46–54, [hep-ph/9709492].

T. Han, T. Yanagida, and R.-J. Zhang, *Adjoint messengers and perturbative unification at the string scale*, *Phys.Rev.* **D58** (1998) 095011, [hep-ph/9804228].

G. Dvali, *Infrared hierarchy, thermal brane inflation and superstrings as superheavy dark matter*, *Phys.Lett.* **B459** (1999) 489–496, [hep-ph/9905204].

K. Hamaguchi, K. Izawa, Y. Nomura, and T. Yanagida, *Longlived superheavy particles in dynamical supersymmetry breaking models in supergravity*, *Phys.Rev.* **D60** (1999) 125009, [hep-ph/9903207].

C. Coriano, A. E. Faraggi, and M. Plumacher, *Stable superstring relics and ultrahigh-energy cosmic rays*, *Nucl.Phys.* **B614** (2001) 233–253, [hep-ph/0107053].

H.-C. Cheng, K. T. Matchev, and M. Schmaltz, *Radiative corrections to Kaluza-Klein masses*, *Phys.Rev.* **D66** (2002) 036005, [hep-ph/0204342].

G. Shiu and L.-T. Wang, *D matter*, *Phys.Rev.* **D69** (2004) 126007, [hep-ph/0311228].

Y. Uehara, JHEP **0112**, 034 (2001).

V. Berezinsky, M. Kachelriess, and M. Solberg, *Supersymmetric superheavy dark matter*, *Phys.Rev.* **D78** (2008) 123535, [arXiv:0810.3012].

T. W. Kephart and Q. Shafi, *Family unification, exotic states and magnetic monopoles*, *Phys.Lett.* **B520** (2001) 313–316, [hep-ph/0105237].

T. W. Kephart, C.-A. Lee, and Q. Shafi, *Family unification, exotic states and light magnetic monopoles*, *JHEP* **0701** (2007) 088, [hep-ph/0602055].

2) We have not conclusively detected a WIMP yet in colliders or in “conventional” direct/indirect detection

i.e. non-thermal DM is **naturally** harder to detect than thermal DM

intuitive: weaker interaction prevents them from thermalizing to begin with.

$\longrightarrow$ Longer we don’t “see” it in conventional experiments, motivation for naturally hidden objects increases.

3) Even a tiny fraction of the total dark matter content in nonthermal DM can be seen through gravitational effects of isocurvature with an O(1) effect.

$$\frac{\delta \rho_X}{\rho_X + \rho_Y} \sim 10^{-4}$$

Even if most of DM were thermal, are we sure that DM is pure to 1 part in a million? If neglected, you will get your CMB fits wrong.

Recap:

Non-thermal dark matter study is well motivated

- mass scales and BSM coupling speculations
- small contamination of DM fraction can be observable

Challenge (opportunity):

need observable handles on the production.

$\boxed{\text{an observable handle: inflation}}$

Theoretical predictivity comparison:

thermal DM	non-thermal DM
insensitivity to production mechanism (i.e. Boltzmann H-theorem)	experimental help: ?? use "inflationary measurements" and other observables sensitive to production
WIMP miracle: weak cross section gives right abundance because $T_0 \sim \frac{m_W^2}{M_p}$	numerological coincidences exist for some candidates: e.g. gravitationally produced superheavy DM (WIMPZILLAs) [DC, Kolb, Riotto 98] $\sqrt{\Gamma M_p} \sim (wavefunction)^{-1} \frac{(2\pi)^3 M_p^2}{H_e m_{DM}} \mathrm{eV}$

Experimental challenge: may not be able to produce them terrestrially.

Part II

What are DM-photon isocurvature perturbations?

DM inhomogeneities different from photons.

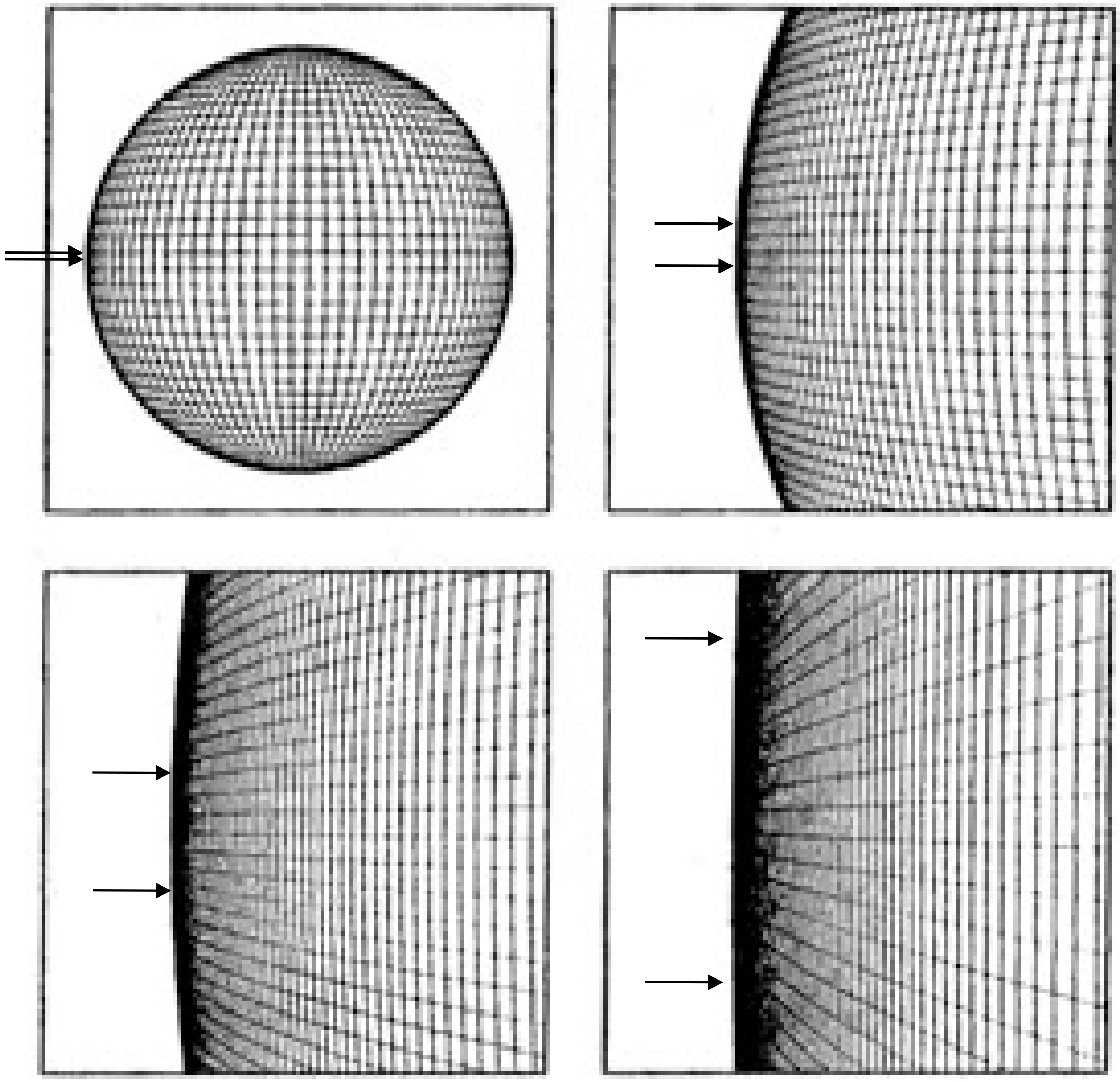
How do you compute them?

$\underbrace{\text{(correlation functions during inflation)}}_{\text{quantum}}$ X (transfer function)

What are fermionic isocurvature perturbations?

Inhomogeneities of non-thermal dark matter fermions that are dynamical during inflation.

inflation is a paradigm for dynamically generating special initial conditions of cosmology

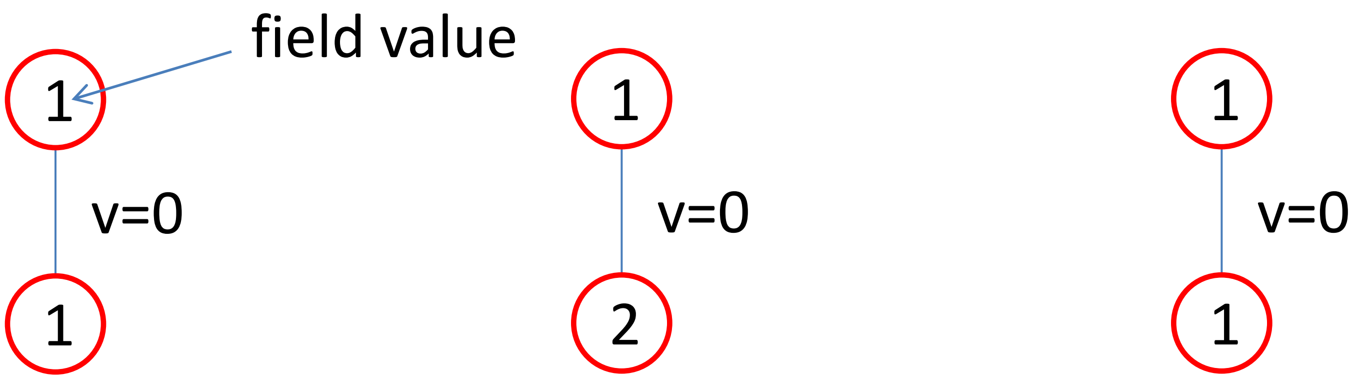


Einstein Eqs.
coupled to
special energy and
pressure stretches
space to make it flat

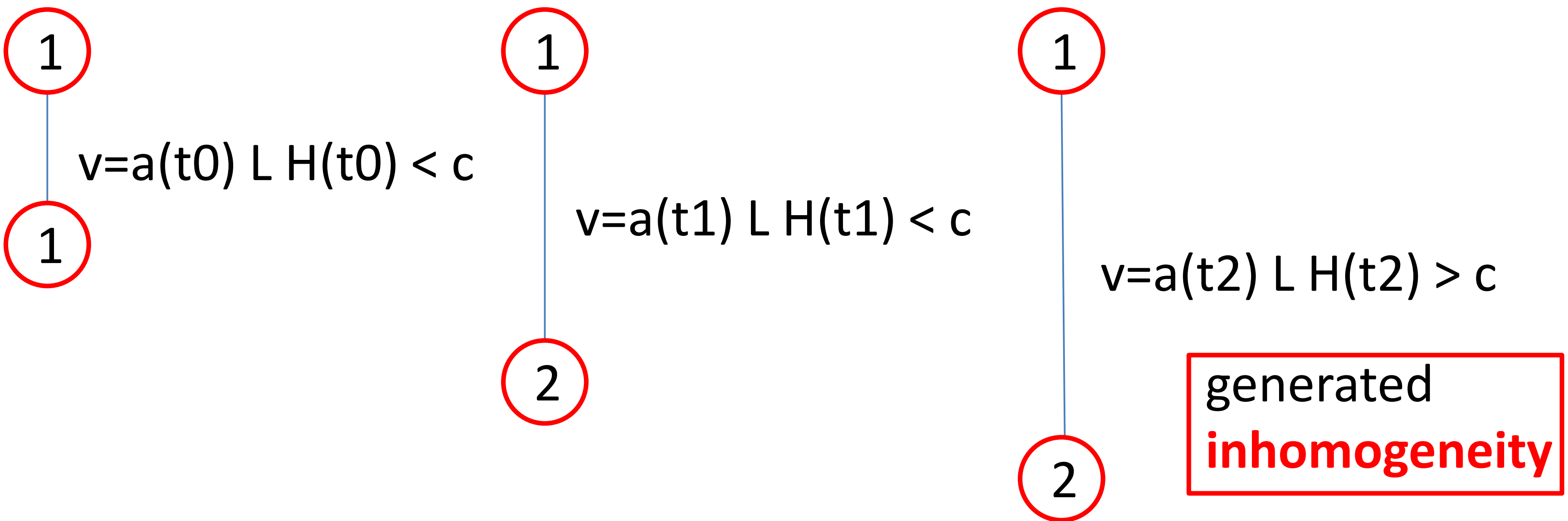
single field

Stretching generates "local average" **inhomogeneities**

flat spacetime **quantum** fluctuations



inflation space-time



One inflationary observable sensitive to non-thermal DM can be partially isocurvature inhomogeneity initial conditions.

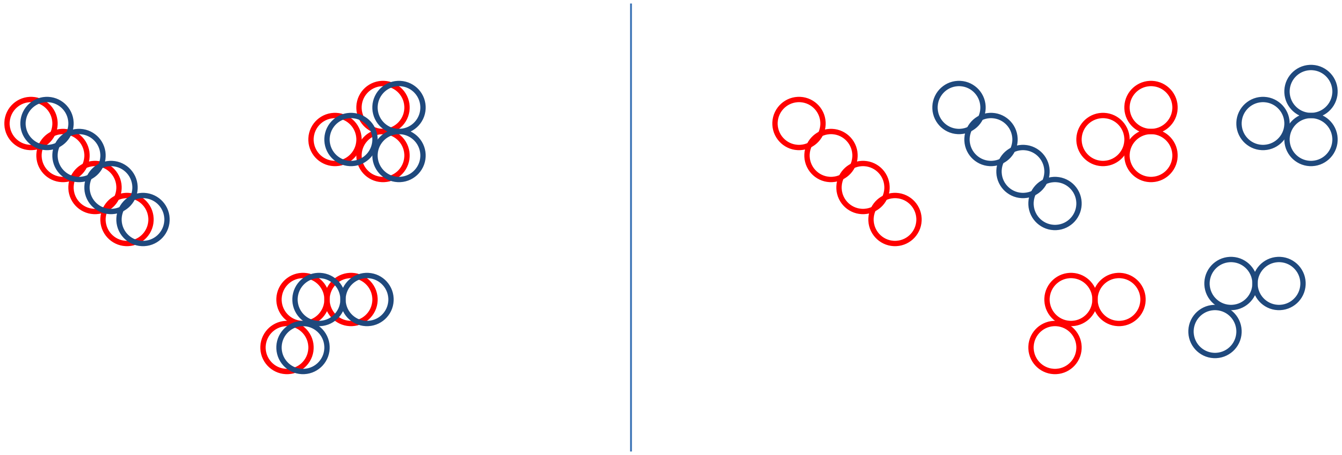
$$S_{DM,\gamma} = \frac{\delta n_{DM}}{n_{DM}} - \frac{\delta n_\gamma}{n_\gamma}$$

adiabatic density inhomogeneity

$$S_{DM,\gamma} = 0$$

isocurvature density inhomogeneity

$$S_{DM,\gamma} \neq 0$$



Special in cosmology: these initial conditions are **time evolution approximate solutions** at early times of relevant Fourier modes of linearized fluid equations, i.e. $S_{DM,\gamma} = \text{constant}$

Single field ϕ inflation cannot generate isocurvature perturbations.

all inhomogeneities are tied to one field

$$\delta\phi(t,\vec{x}) \to \delta n_X(t,\vec{x})$$

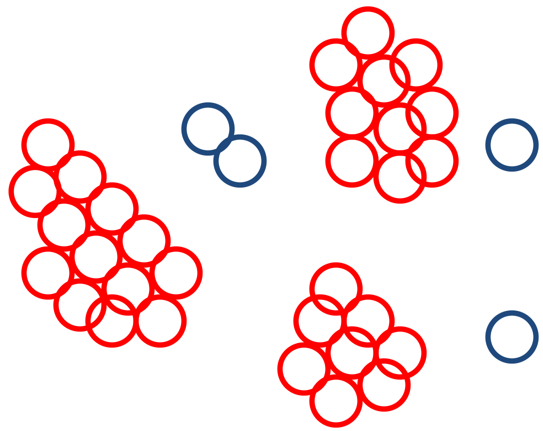
$$\delta\phi(t,\vec{x}) \to \delta n_\gamma(t,\vec{x})$$

N. B. The physical solution is continuously connected by k to a pure diffeomorphism gauge mode.

[clearest reference is Weinberg 04]

Multiple dynamical degrees of freedom naturally lead to isocurvature initial conditions in fields which the inflaton field and its decay products are very weakly coupled to (e.g. **non-thermal DM candidates**)

$$\delta\phi(t,\vec{x}) \rightarrow \delta n_{\gamma}(t,\vec{x})$$



$$\delta X(t,\vec{x}) \rightarrow \delta n_{DM}(t,\vec{x})$$

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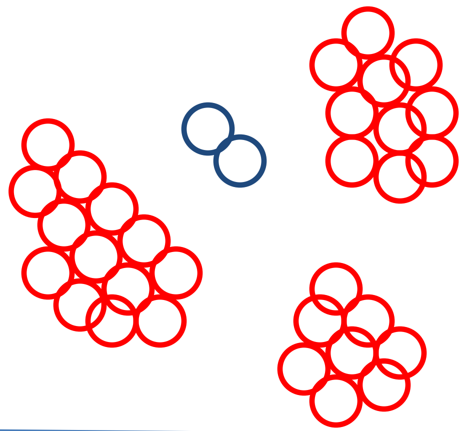
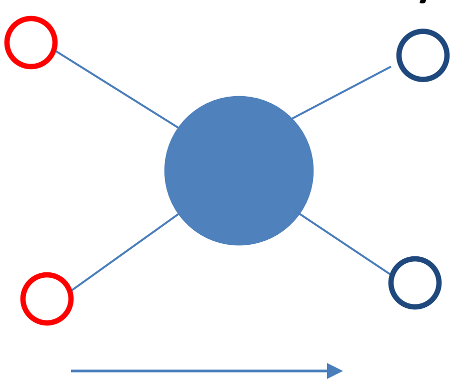


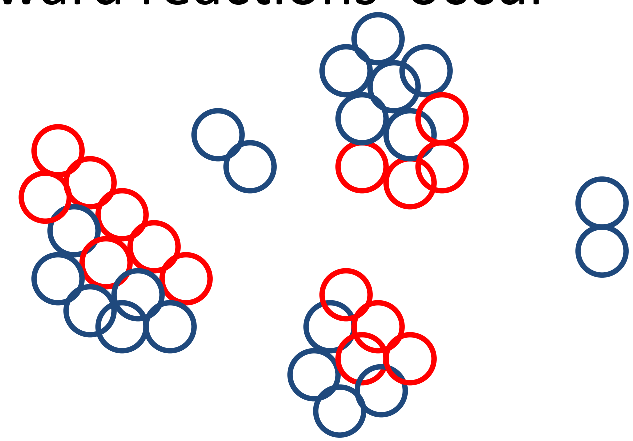
$$\delta X(t,\vec{x}) \rightarrow \delta n_{DM}(t,\vec{x})$$

If there are many more than , and forward reactions occur

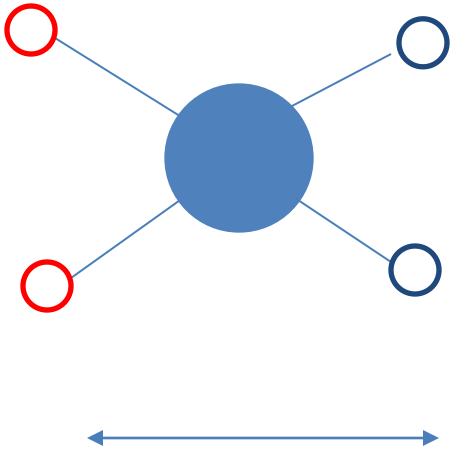


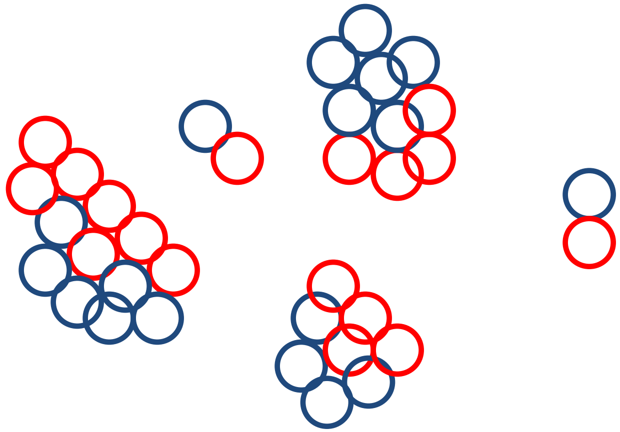
makes enough blue s.t.

adiabatic again



If thermal equilibrium, then





adiabatic again since inhomogeneity is the same.

[e.g. Weinberg 04]

The cosmological effect of CDM isocurvature on cosmology is primarily gravitational.

Newtonian intuition:

$$(\text{equation governing potential}) = \frac{1}{M_p^2}(\delta\rho_m + \delta\rho_\gamma)$$

Potential force and pressure push fluid components such as $\delta\rho_\gamma$ around.

e.g.

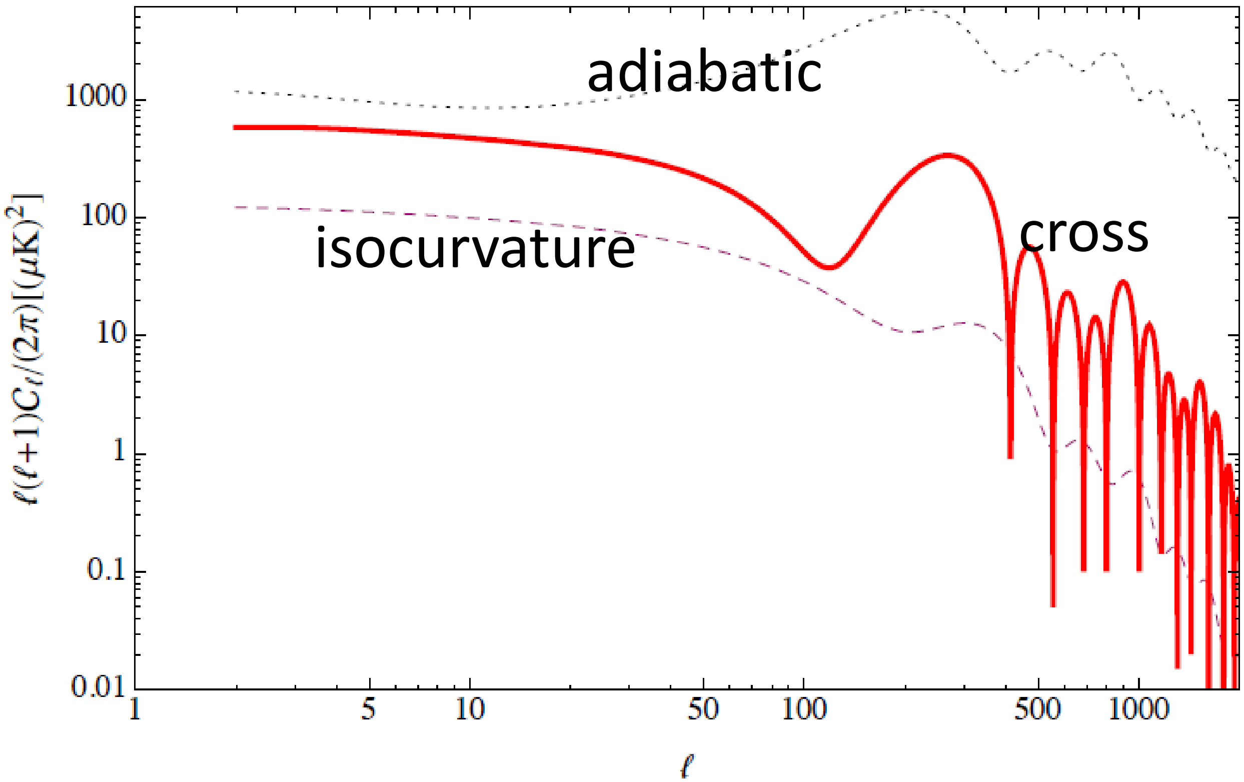
$$\frac{\delta\rho_\gamma}{\rho_\gamma}(k_s,\eta) \sim \Phi(k_s,\eta_i)\sqrt{c_s}\cos\left(k_s\int^{\eta} c_s(\eta')d\eta'\right) - S^i_{DM,\gamma}(k_s,\eta_i)\frac{\sin\left(k\eta/\sqrt{3}\right)}{k_s\eta_{eq}} \qquad k_s \gg k_{eq}$$

This makes CMB sensitive primarily $\boxed{\text{on large scales.}}$

CMB

Scale invariant isocurvature

$\beta = -1$



$$\boxed{\alpha|_{\beta=0} < 0.016 \text{ (95\% CL) and } \alpha|_{\beta=-1} < 0.0011 \text{ (95\% CL)}}$$

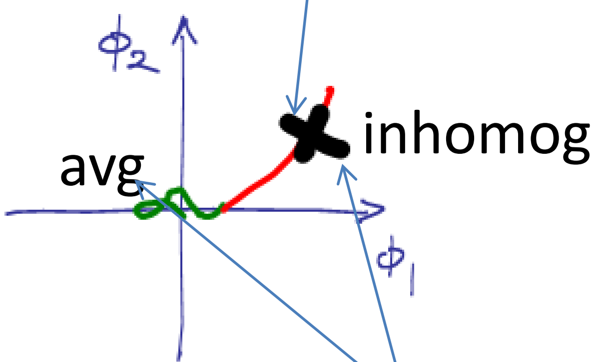
Planck+WP 2013

In general, final observable effects are

(inhomogeneity [parameters]) X (avg energy density [parameters])

different models give different correlations

isocurvature



multi-field

pre-2013 isocurvature DM candidates

- axions
- Affleck-Dine
- SUSY/String theory moduli
- WIMPZILLAs

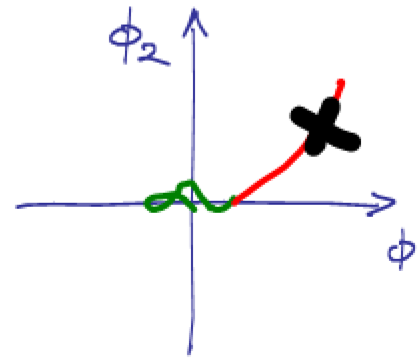
"particles produced" after quantum statistics generated

All previously known models were bosonic.

next: a new class of scenarios – **fermionic isocurvature** perturbations

Fermion Quantum Fluctuations During Inflation

single field

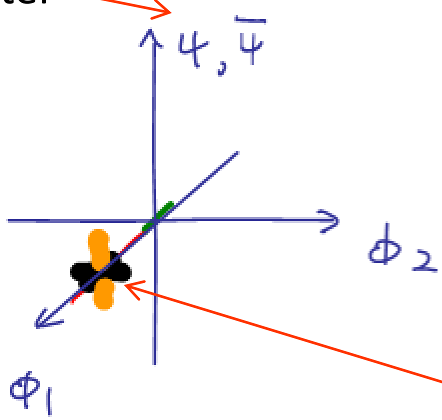


multi-field

What about fermion field fluctuations during inflation?

DC, Yoo, Zhou 1306.1966

e.g. dark matter from hidden sector



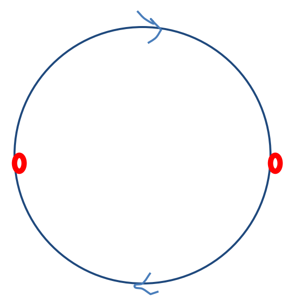
Observable: $\langle \delta T^{(i)}_{\alpha\beta}(t,\vec{x})\delta T^{(j)}_{\mu\nu}(t,\vec{y})\ldots\rangle$

no classical VEV for fermions is an important generic character of this class of isocurvature perturbations

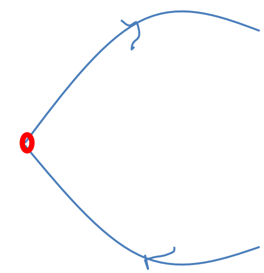
multi-field + fermion

avg density in fermionic isocurvature:

flat spacetime



inflationary spacetime

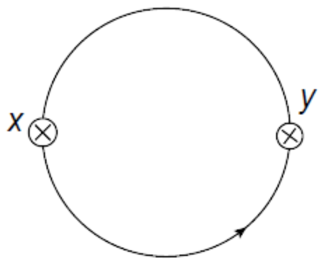


Heisenberg uncertainty + expansion work +
conformal symmetry breaking through mass term →

Hawking radiation in inflationary background.

next, inhomogeneities

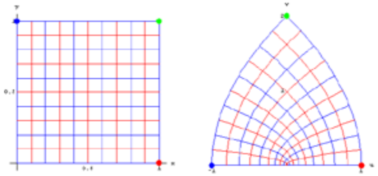
Consider "non-interacting" fermion quantum inhomogeneity during inflation.



Gives a very **blue** exponentially negligible inhomogeneity

reason: A special symmetry of massless fermions called Weyl invariance (at tree level)

spin 1/2



$$g_{\mu\nu}(x) \to \lambda(x) g_{\mu\nu}(x) \qquad \Psi \to \lambda^{-3/2}(x)\Psi$$

$$\sqrt{g} i \bar{\Psi} \gamma^a \nabla_{e_a} \Psi \to \sqrt{g} i \bar{\Psi} \gamma^a \nabla_{e_a} \Psi$$

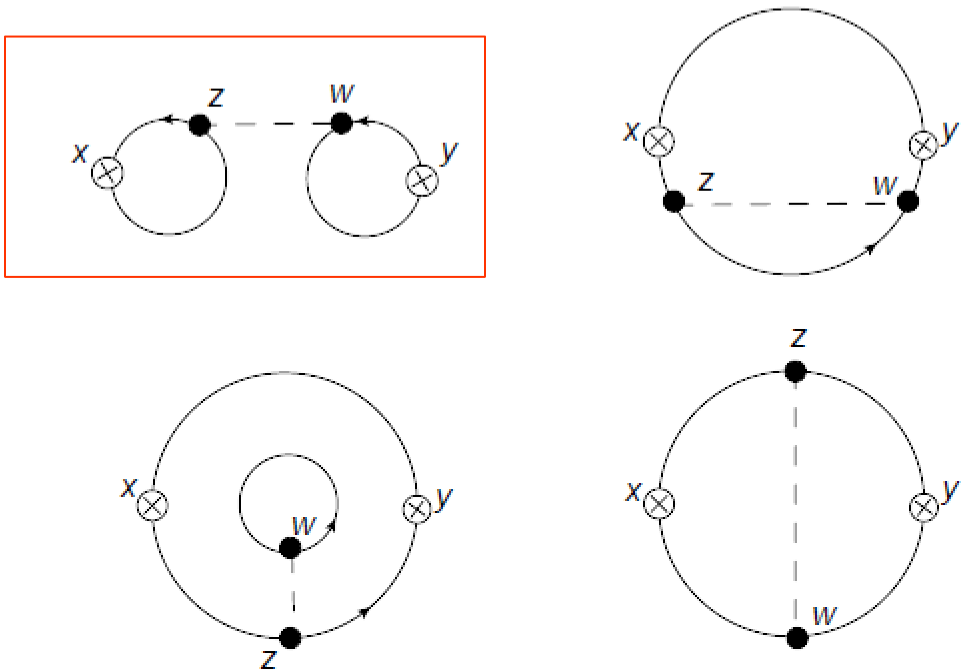
Fermionic isocurvature perturbations [DC, Yoo, Zhou 13]

Need to add interactions:

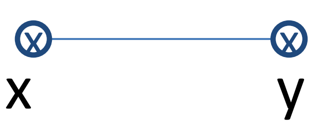
add Yukawa interaction among fermions

intuition: $V_{int} = -\lambda^2 \frac{e^{-m_\sigma r}}{r}$

dominates



Contrast to axions:



Recall:

In general, final observable effects are

(inhomogeneity [parameters]) X (avg energy density [parameters])

different models give different correlations

Isocurvature 2-pt Comparison

	$\Delta^2_{\delta_S}$	$\Omega_X h^2 (\propto \omega_X)$
fermion (in the computationally simple regime) $m_\psi > \lambda H_{\rm inf}/2\pi$ [1306.1966]	$\omega_\psi^2 \frac{\lambda^2 H^2(t_1)}{4\pi^2 m_\psi^2}\left(\frac{k}{a(t_1)H(t_1)}\right)^{-2\epsilon}$	$\left(\frac{m_\psi}{10^{11}\mathrm{GeV}}\right)^2 \left(\frac{T_{RH}}{10^9\mathrm{GeV}}\right)$
axion [e.g. ph/0606107]	$\omega_a^2 \frac{H^2(t_1)}{\pi^2 f_a^2 \langle\theta^2_{QCD}\rangle}\left(\frac{k}{a(t_1)H(t_1)}\right)^{-2\epsilon}$	$7.24\, g_{*,1}^{-5/12} \langle\Theta^2_{\rm QCD}\rangle \left(\frac{200\mathrm{MeV}}{\Lambda_{\rm QCD}}\right)^{\frac{3}{4}} \left(\frac{1\mu\mathrm{eV}}{m_a}\right)^{\frac{7}{6}}$

tunable diff from axions

axions are initial condition sensitive

Fermions have different parametric relationship between $\Delta^2_{\delta_S}$ $\Omega_X h^2 (\propto \omega_X)$

Isocurvature 2-pt Comparison

$\Delta^2_{\delta_S}$ $\Omega_X h^2 (\propto \omega_X)$

tunable diff from axions

fermion (in the computationally simple regime) $m_\psi > \lambda H_{\mathrm{inf}}/2\pi$ [1306.1966]

$$\omega_\psi^2 \frac{\lambda^2 H^2(t_1)}{4\pi^2 m_\psi^2}\left(\frac{k}{a(t_1)H(t_1)}\right)^{-2\epsilon} \qquad \left(\frac{m_\psi}{10^{11}\mathrm{GeV}}\right)^2\left(\frac{T_{RH}}{10^9\mathrm{GeV}}\right)$$

axion [e.g. ph/0606107]

$$\omega_a^2 \frac{H^2(t_1)}{\pi^2 f_a^2 \langle\theta^2_{QCD}\rangle}\left(\frac{k}{a(t_1)H(t_1)}\right)^{-2\epsilon} \qquad 7.24\, g_{*,1}^{-5/12}\langle\Theta^2_{\mathrm{QCD}}\rangle\left(\frac{200\mathrm{MeV}}{\Lambda_{\mathrm{QCD}}}\right)^{\frac{3}{4}}\left(\frac{1\mu\mathrm{eV}}{m_a}\right)^{\frac{7}{6}}$$

axions are initial condition sensitive

Fermions have different parametric relationship between $\Delta^2_{\delta_S}$ $\Omega_X h^2 (\propto \omega_X)$

In the case of axion, isocurvature bound + all relic abundance from axion + PQ broken during inflation + no symmetry restoration:

$$H(t_1) \lesssim \begin{cases} 10^7\ \mathrm{GeV}\left(\frac{f}{10^{11}\ \mathrm{GeV}}\right)^{0.4} & f \lesssim 10^{17}\ \mathrm{GeV} \\ 10^8\ \mathrm{GeV}\left(\frac{f}{10^{11}\ \mathrm{GeV}}\right)^{1/4} & f \gtrsim 10^{17}\ \mathrm{GeV} \end{cases}$$

[e.g. 1303.5082, 1403.4594]

BICEP 2 (in slow-roll inflation): $H(t_1) = 6 \times 10^{14}\ \mathrm{GeV}$

$\longrightarrow$ $f \gg M_{pl}$ minimal axions of this type looks bad

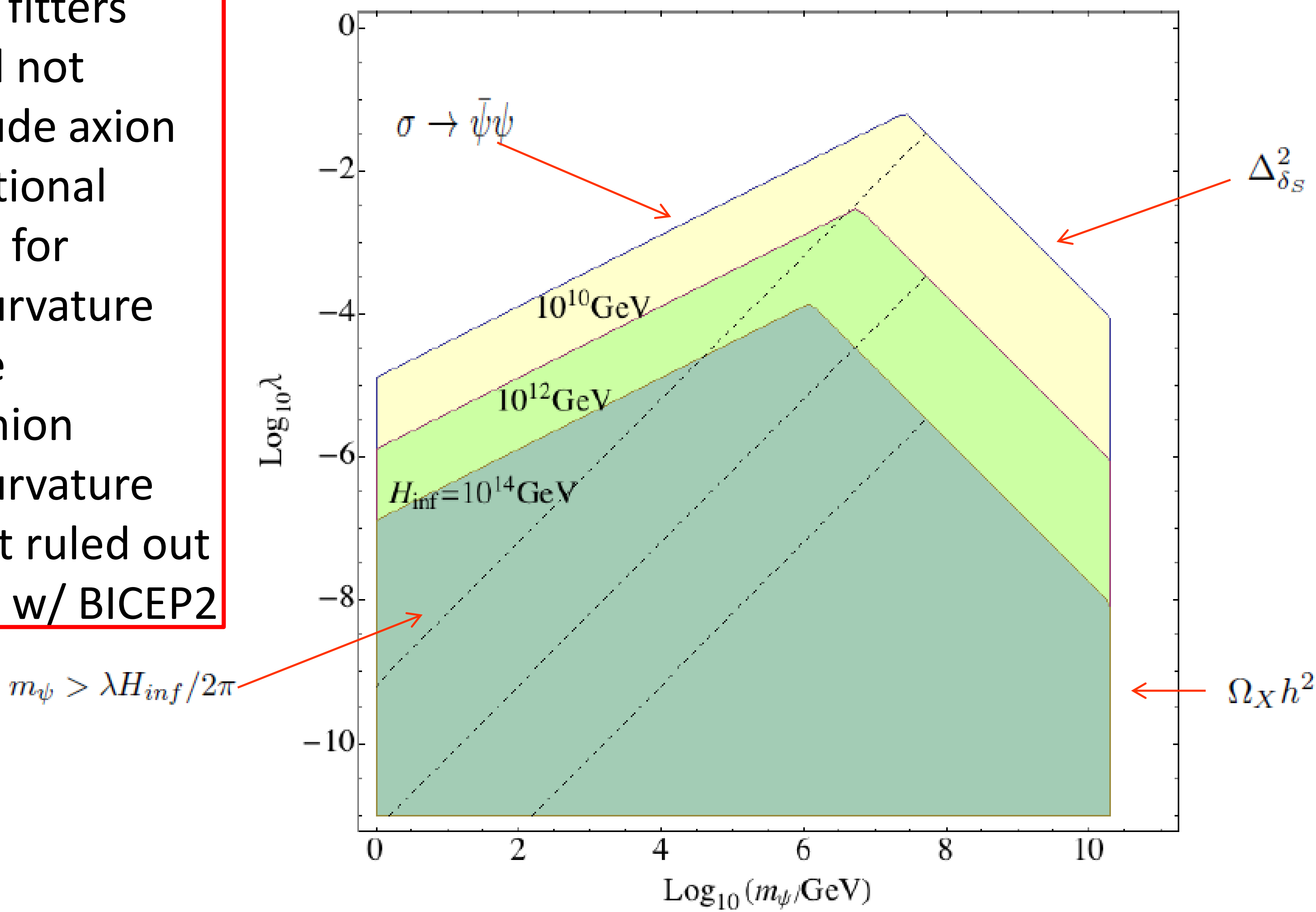
[e.g. 1403.4186, 1403.4594]

fermion isocurvature survives BICEP2

Allowed Parameter Region

$T_{RH} = 10^9 GeV$

Data fitters need not exclude axion functional form for isocurvature since ferimion isocurvature is not ruled out even w/ BICEP2



[DC, Yoo, Zhou 13]

Aside: Our work is the first fermionic isocurvature perturbation computation in the literature.

answers "What imprints come from fermion fluctuations during inflation?"

interesting reference at the technical level:

requires composite operator curved spacetime renormalization with 8 counter-terms:

$$(\bar{\psi}\psi)_{x,r} = (\bar{\psi}_x)_r(\psi_x)_r(1+\delta Z_1) + \delta Z_2(\sigma_{x,r})^3 + \delta Z_3(\sigma_{x,r})^2 + \delta Z_4\sigma_{x,r} + \delta Z_5 + \delta Z_6\Box\sigma_{x,r} + \delta Z_7 R + \delta Z_8 R\sigma_{x,r}$$

First explicit computation of this kind in inflation.

Computation relies crucially on diffeomorphism Ward identities.

Computation requires consideration of physics of both quasi-dS space **and the transition to post inflationary space**.

Admixture sensitivity is typical of isocurvature spectrum

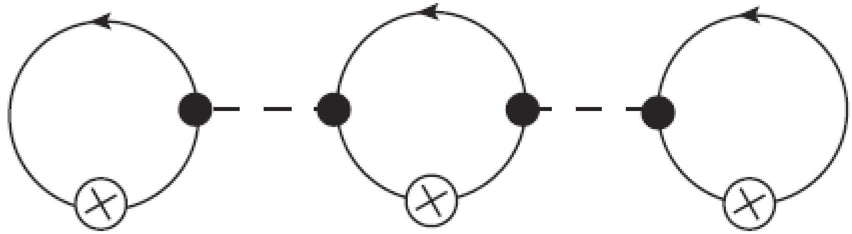
$$\Delta^2_{\delta_S} = \omega^2_\psi \frac{\lambda^2 H^2(t_1)}{4\pi^2 m^2_\psi}\left(\frac{k}{a(t_1)H(t_1)}\right)^{-2\epsilon}$$

$$\omega_\psi = \frac{\rho_\psi}{\rho_\psi + \rho_X}$$

current data: $\Delta^2_{\delta_S}$ < 3% of $\Delta^2_\zeta \approx 2 \times 10^{-9}$

It is sensitive to ω_ψ as small as 10^{-5}

3-Point Function



$$\boxed{B_S(\vec{p}_1, \vec{p}_2, \vec{p}_3) \; = \; \lambda^4 \omega_\psi^3 \frac{(\partial_m n_\psi)^2 \, (\partial_m^2 n_\psi)}{n_\psi^3} \left[\Delta_\sigma^2(p_1)\Delta_\sigma^2(p_2) + 2 \text{ perms}\right]}$$

non-gaussianities are similar to the local type

$$f_{NL}^S \; \sim \; a_1 \left(\frac{\alpha_S(\lambda, m_\psi, H_e, T_{RH})}{0.02}\right)^2 \left(\frac{\Omega_\psi h^2(m_\psi, T_{RH})}{10^{-7}}\right)^{-1} \left(\frac{m_\psi/H_e}{10^{-1}}\right)$$

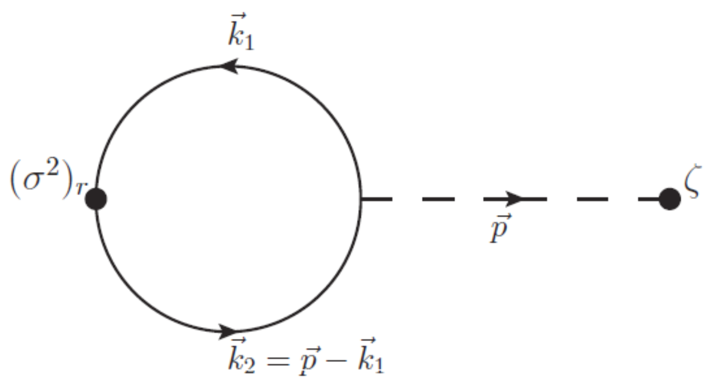
Can be observationally large in a corner of the parameter space.

Are Axions Cross-Correlated w/ Curvature?

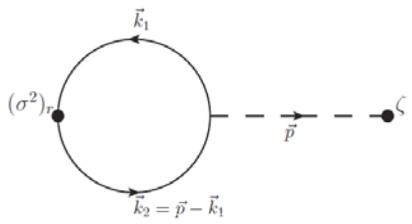
***Planck* 2013 results. XXII. Constraints on inflation** [1303.5082]

Within the general parametrization presented in Eq. 66, we can select the axion case by imposing $\boxed{\mathcal{P}^{(1,2)}_{\mathcal{RI}} = 0}$ as well as the

Of course, this cannot be strictly true for $\langle\sigma\rangle = 0$



axions are cross correlated with curvature perturbations



$$\beta \equiv -\frac{\Delta^2_{\zeta\delta_S}(k)}{\sqrt{\Delta^2_\zeta(k)\Delta^2_{\delta_S}(k)}}$$

Naïve analysis of this quantity can allow this quantity to be O(1). Gauge symmetry of general relativity leads to cancellations giving a small, but non-zero quantity.

$$\left| \beta_{axion} = -\frac{\sqrt{\Delta^2_\zeta}}{2}\left(\ln\frac{p}{\Lambda_{IR}}\right)^{-1/2} \right| \lesssim 2.5 \times 10^{-5}$$

IR sensitive

[DC, Yoo, Zhou 13]

$\boxed{\text{If measured, it is a test of gauge symmetry of GR.}}$

Part III

What is something interesting for the future in this area?

$\boxed{\text{blue spectrum}}$

In addition to digging below the 3% level, what else may be interesting?

Since isocurvature spectrum need not be scale invariant, a blue spectrum can have large amplitudes on short length scales while completely hidden on long CMB scales.

matter power spectra

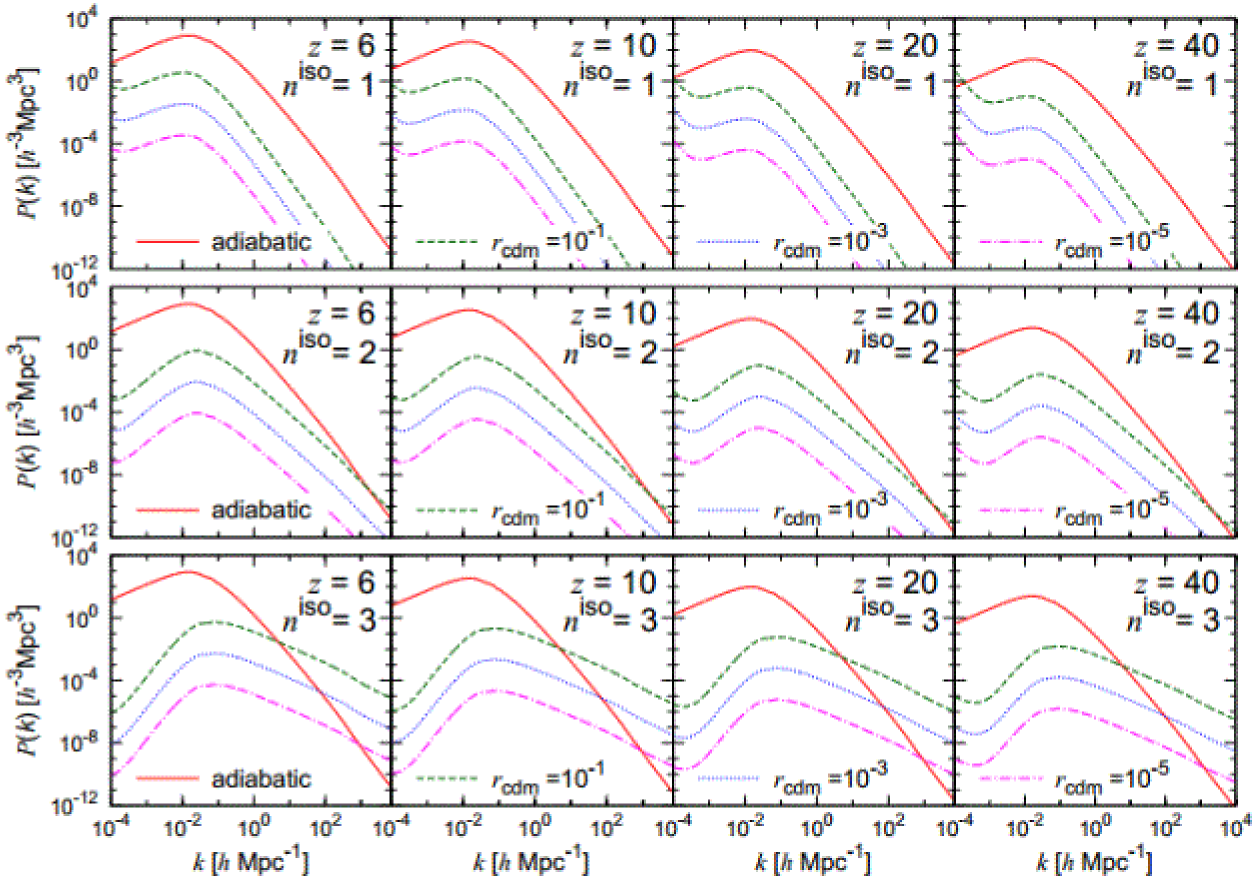


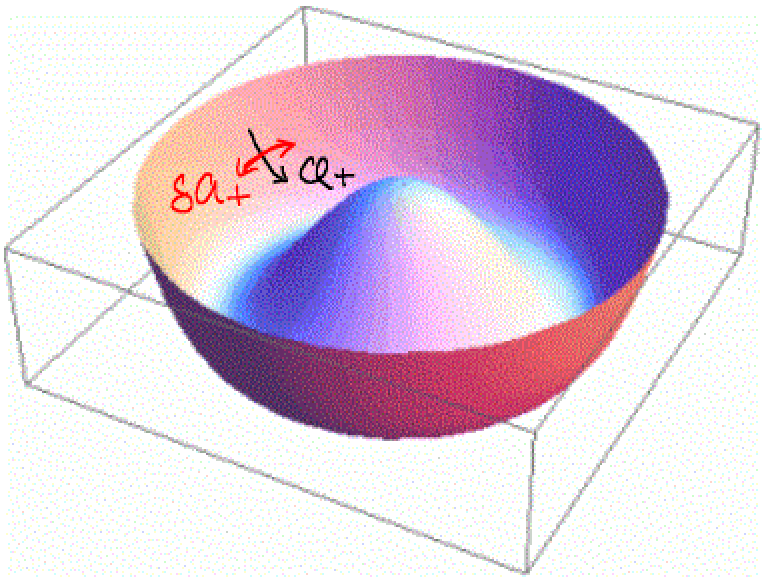
Figure 1. The matter power spectra generated by adiabatic or pure CDM isocurvature fluctuations, at redshifts $z = 6$, 10, 20 and 40 (from left column to right). The spectral indices of the isocurvature mode are as $n_s^{\rm iso} = 1$, 2 and 3 (from top row to bottom). In each panel, the different curves represent the matter power spectrum of the adiabatic fluctuations (solid/red) and the CDM isocurvature fluctuations with $r_{\rm cdm} = 10^{-1}$ (dashed/green), 10^{-3} (dotted/blue) and 10^{-5} (dot-dashed/magenta). The isocurvature spectra shown have no contribution from adiabatic fluctuations.

[Takeuchi, Chongchitnan 13]

example

$$W = h(\Phi_+\Phi_- - F_a^2)\Phi_0$$

$$\delta\theta_+ \equiv \frac{\delta a_+}{\varphi_+}$$



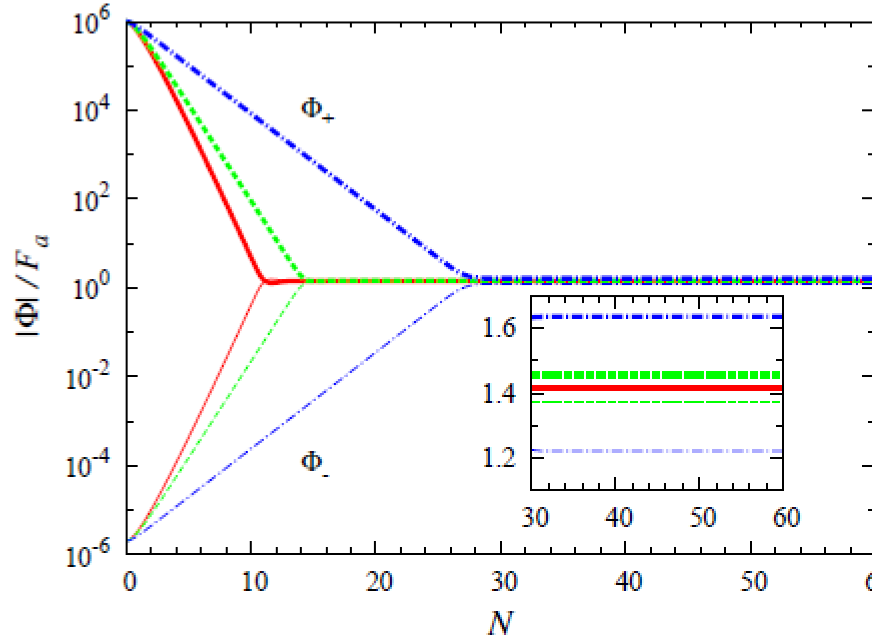
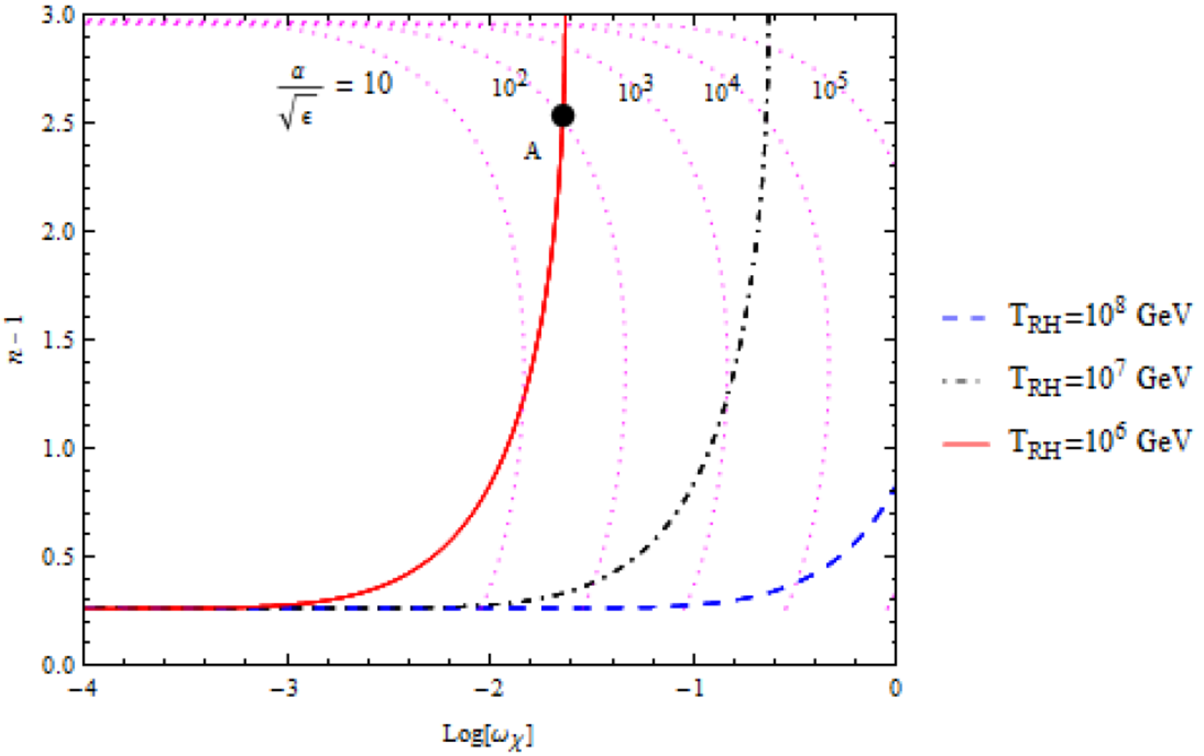


FIG. 1: Evolution of the fields Φ_+ (upper thick lines) and Φ_- (lower thin lines) for $c_- = 9/4$ and $c_+ = 9/4$ ($n_{\rm iso} = 4$, red solid), 2 ($n_{\rm iso} = 3$, green dashed), and 5/4 ($n_{\rm iso} = 2$, blue dotted-dashed). The inset shows the minima where the fields settle down.

[Kasuya, Kawasaki 09]

Some **preliminary** results:

SUGRA theories have an eta problem. This can naturally generate a blue spectrum.



[DC et al. preliminary]

Future:

• Space of models with blue spectrum?

What can we learn about high energy theory if
we measure blue spectrum? Perhaps SUGRA scenarios
(in progress) What else?

• What kind of discovery probes are possible with
a blue spectrum?

• What are the constraints on plausible reheating scenarios
coming from isocurvature constraints?

Summary

- Non-thermal dark matter is theoretically well motivated.

- Non-thermal dark matter needs more observables than thermal WIMPs for over-constraint.

- Inflationary production can be probed by isocurvature perturbations.

- Fermionic isocurvature perturbations are a new class of isocurvature perturbations where interactions are seen in the sky.

- Interesting work in blue spectra may remain.